Integrales indefinidas

11

ACTIVIDADES

1. Página 266

$f(x)=5x^4-3x^2+2 \longrightarrow F(x)=x^5-x^3+2x+3$

$f(x)=x^4+x^3-2 \longrightarrow F(x)=\frac{x^5}{5}+\frac{x^4}{4}-2x+1$

$f(x)=12x^2+6x \longrightarrow F(x)=4x^3+3x^2$

2. Página 266

a) $2x^4-2x^2+5x$

b) $\frac{2}{5}x^4$

c) $3e^x+x$

d) $3\sqrt{x}$

3. Página 267

a) $\int(3x^2-2x+1)dx=x^3-x^2+x+k$

b) $\int(2\cos x)dx=2\text{sen}\,x+k$

c) $\int(4x^3-\text{sen}\,x)dx=x^4+\cos x+k$

d) $\int(2x-e^x)dx=x^2-e^x+k$

e) $\int(3-e^{-x})dx=3x+e^{-x}+k$

f) $\int(5-4x-\cos x)dx=5x-2x^2-\text{sen}\,x+k$

g) $\int(5e^x+2\cos x)dx=5e^x+2\text{sen}\,x+k$

h) $\int(7\text{sen}\,x+4\cos x)dx=-7\cos x+4\text{sen}\,x+k$

4. Página 267

a) $\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx=F(x)+G(x)+k$

b) $\int[2f(x)-g(x)]dx=2\int f(x)dx-\int g(x)dx=2F(x)-G(x)+k$

c) $\int\left[\frac{1}{2}f(x)-2g(x)\right]dx=\frac{1}{2}\int f(x)dx-2\int g(x)dx=\frac{1}{2}F(x)-2G(x)+k$

d) $\int[-f(x)+b\cdot g(x)]dx=-\int f(x)dx+b\cdot\int g(x)dx=-F(x)+b\cdot G(x)+k$

5. Página 268

a) $\int(x^2+3x-2)dx=\frac{1}{3}x^3+\frac{3}{2}x^2-2x+k$

b) $\int\frac{3}{x^3}dx=\int 3x^{-3}dx=-\frac{3}{2x^2}+k$

c) $\int 3\sqrt[4]{x^3}dx=\int 3x^{\frac{3}{4}}dx=\frac{12}{7}\sqrt[4]{x^7}+k=\frac{12}{7}x\sqrt[4]{x^3}+k$

6. Página 268

a) $\int (1-x)^2 dx = -\int (-1)(1-x)^2 dx = -\frac{(1-x)^3}{3} + k$

b) $\int (1-x^2)^2 dx = \int (1-2x^2+x^4)dx = x - \frac{2}{3}x^3 + \frac{x^5}{5} + k$

c) $\int 2x\sqrt{x^2+3}\,dx$ $\rightarrow$ Tomamos $f(x) = x^2+3 \rightarrow f'(x) = 2x$ $\rightarrow$ $\int 2x\sqrt{x^2+3}dx = \frac{2}{3}\sqrt{(x^2+3)^3} + k$

7. Página 269

a) $\int x(x^2+3)dx = \frac{1}{2}\int 2x(x^2+3)dx = \frac{1}{2}\cdot\frac{(x^2+3)^2}{2} + k = \frac{(x^2+3)^2}{4} + k$

b) $\int (x-2)(x^2-4x+1)^3 dx = \frac{1}{2}\int (2x-4)(x^2-4x+1)^3 dx = \frac{1}{2}\cdot\frac{(x^2-4x+1)^4}{4} + k = \frac{(x^2-4x+1)^4}{8} + k$

8. Página 269

a) $\int \sqrt{2x-1}\,dx = \frac{1}{2}\int 2\sqrt{2x-1}\,dx = \frac{1}{2}\cdot\frac{2}{3}\sqrt{(2x-1)^3} + k = \frac{\sqrt{(2x-1)^3}}{3} + k$

b) $\int x\sqrt{x^2-2}dx = \frac{1}{2}\int 2x\sqrt{x^2-2}dx = \frac{1}{2}\cdot\frac{2}{3}\sqrt{(x^2-2)^3} + k = \frac{\sqrt{(x^2-2)^3}}{3} + k$

c) $\int e^{2x}dx = \frac{1}{2}\int 2e^{2x}dx = \frac{1}{2}e^{2x} + k$

d) $\int (x+1)e^{x^2+2x-1}dx = \frac{1}{2}\int (2x+2)e^{x^2+2x-1}dx = \frac{1}{2}e^{x^2+2x-1} + k$

9. Página 270

a) $\int \frac{4x}{2x^2+1}dx = \ln|2x^2+1| + k$

b) $\int \frac{x^2}{x^3+3}dx = \frac{1}{3}\int \frac{3x^2}{x^3+3}dx = \frac{1}{3}\ln|x^3+3| + k$

c) $\int \frac{x+1}{x^2+2x}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x}dx = \frac{1}{2}\ln|x^2+2x| + k$

d) $\int \frac{5x}{1-x^2}dx = -\frac{5}{2}\int \frac{-2x}{1-x^2}dx = -\frac{5}{2}\ln|1-x^2| + k$

10. Página 270

a) $\int \frac{4x^3}{x^4+2}dx = \ln|x^4+2| + k$

b) $\int \frac{4x^3}{(x^4+2)^2}dx = \int 4x^3(x^4+2)^{-2}dx = \frac{-1}{x^4+2} + k$

c) $\int \frac{4x^3}{\sqrt[3]{x^4+2}}dx = \int 4x^3(x^4+2)^{-\frac{1}{3}}dx = \frac{3}{2}\sqrt[3]{(x^4+2)^2} + k$

d) $\int \frac{x-1}{\sqrt{x^2-2x}}dx = \frac{1}{2}\int (2x-2)\cdot(x^2-2x)^{-\frac{1}{2}}dx = \frac{1}{2}\cdot 2\cdot(x^2-2x)^{\frac{1}{2}} + k = \sqrt{x^2-2x} + k$

11. Página 271

a) $\int 3^{\frac{x}{2}}dx = 2\cdot\int \frac{1}{2}\cdot 3^{\frac{x}{2}}dx = 2\cdot\frac{3^{\frac{x}{2}}}{\ln 3}+k$

b) $\int e^{x+1}dx = e^{x+1}+k$

c) $\int \left(\frac{1}{2}\right)^{4x}dx = \frac{1}{4}\int 4\cdot\left(\frac{1}{2}\right)^{4x}dx = \frac{1}{4}\cdot\frac{\left(\frac{1}{2}\right)^{4x}}{\ln\left(\frac{1}{2}\right)}+k$

d) $\int (e^{-3x}+e^{x-2})dx = \int e^{-3x}dx + \int e^{x-2}dx = -\frac{1}{3}\int -3e^{-3x}dx + \int e^{x-2}dx = -\frac{1}{3}e^{-3x}+e^{x-2}+k$

12. Página 266

a) $\int 7^{x^2+1}\cdot 2xdx = \frac{7^{x^2+1}}{\ln(7)}+k$

b) $\int 5e^{\frac{x}{2}+2}dx = 10\int \frac{1}{2}e^{\frac{x}{2}+2}dx = 10e^{\frac{x}{2}+2}+k$

c) $\int \frac{3^{5x-1}}{7}dx = \frac{1}{35}\int 5\cdot 3^{5x-1}dx = \frac{1}{35}\cdot\frac{3^{5x-1}}{\ln(3)}+k$

d) $\int \frac{x}{e^{x^2}}dx = -\frac{1}{2}\int -2xe^{-x^2}dx = -\frac{1}{2}e^{-x^2}+k$

13. Página 272

a) $\int sen(2x)dx = \frac{1}{2}\int 2\cdot sen(2x)dx = -\frac{1}{2}cos(2x)+k$

b) $\int cos(x+1)dx = sen(x+1)+k$

c) $\int \frac{sen\frac{x}{2}}{2}dx = -cos\frac{x}{2}+k$

d) $\int sen(-x)dx = cos(-x)+k$

14. Página 272

a) $\int \frac{1}{cos^2(x+1)}dx = tg(x+1)+k$

b) $\int -3sen(2x+1)dx = -\frac{3}{2}\int 2sen(2x+1)dx = \frac{3}{2}cos(2x+1)+k$

c) $\int (x+1)\cdot cos(x^2+2x)dx = \frac{1}{2}\int (2x+2)\cdot cos(x^2+2x)dx = \frac{1}{2}sen(x^2+2x)+k$

d) $\int \frac{x}{cos^2(x^2-3)}dx = \frac{1}{2}\int \frac{2x}{cos^2(x^2-3)}dx = \frac{1}{2}tg(x^2-3)+k$

15. Página 273

a) $\int \frac{1}{\sqrt{1-25x^2}}dx = \frac{1}{5}arc\,sen(5x)+k$

b) $\int \frac{1}{1+(x-3)^2}dx = arc\,tg(x-3)+k$

16. Página 273

a) $\int \frac{1}{\sqrt{1-(2x-3)^2}}dx = \frac{1}{2}\int \frac{2}{\sqrt{1-(2x-3)^2}}dx = \frac{1}{2}arcsen(2x-3)+k$

b) $\int \frac{x}{1+9x^4}dx = \frac{1}{6}\int \frac{6x}{1+(3x^2)^2}dx = \frac{1}{6}arctg(3x^2)+k$

17. Página 274

a) $\int (x^2+x)e^{-2x+1}dx = (x^2+x)\left(-\frac{1}{2}e^{-2x+1}\right)+\int \frac{1}{2}e^{-2x+1}\cdot(2x+1)dx =$

$u = x^2 + x \rightarrow du = (2x +$

$u = 2x+1 \rightarrow du = 2dx$

$dv = e^{-2x+1}dx \rightarrow v = -\frac{1}{2}e$

$= (x^2+x)\left(-\frac{1}{2}e^{-2x+1}\right)+\frac{1}{2}\left[(2x+1)\left(-\frac{1}{2}\right)e^{-2x+1}-\int\left(-\frac{1}{2}e^{-2x+1}\right)\cdot 2dx\right]=$

$= -\frac{1}{2}(x^2+x)(e^{-2x+1})+\frac{1}{2}\left[-\frac{1}{2}(2x+1)e^{-2x+1}-\frac{1}{2}e^{-2x+1}\right]+k=$

$= -\frac{1}{2}e^{-2x+1}x^2-\frac{1}{2}e^{-2x+1}x-\frac{1}{2}e^{-2x+1}x-\frac{1}{4}e^{-2x+1}-\frac{1}{4}e^{-2x+1}+k = -\frac{1}{2}e^{-2x+1}x^2-e^{-2x+1}x-\frac{1}{2}e^{-2x+1}+k=$

$= \left(-\frac{1}{2}x^2-x-\frac{1}{2}\right)e^{-2x+1}+k$

b) $\int x^2\cdot cos(3x)dx = x^2\cdot\frac{1}{3}sen(3x)-\int\frac{1}{3}sen(3x)\cdot 2xdx =$

$u = x^2 \rightarrow du = 2xdx$

$dv = cos(3x)dx \rightarrow v = \frac{1}{3}se$

$u = x \rightarrow du = dx$

$dv = sen(3x)dx \rightarrow v = -\frac{1}{3}co$

$= \frac{1}{3}x^2sen(3x)-\frac{2}{3}\left[-\frac{1}{3}x\cdot cos(3x)+\frac{1}{3}\int cos(3x)dx\right]=$

$= \frac{1}{3}x^2sen(3x)+\frac{2}{9}xcos(3x)-\frac{2}{27}sen(3x)+k = \left(\frac{1}{3}x^2-\frac{2}{27}\right)sen(3x)+\frac{2}{9}x\,cos(3x)+k$

18. Página 274

a) $\int 2x^2\cdot \ln x dx = \frac{2}{3}x^3\ln x-\int\frac{2}{3}x^3\cdot\frac{1}{x}dx = \frac{2}{3}x^3\ln x-\frac{2}{3}\int x^2dx = \frac{2}{3}x^3\ln x-\frac{2}{3}\int x^2dx = \frac{2}{3}x^3\ln x-\frac{2}{9}x^3+k$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = 2x^2dx \rightarrow v = \frac{2}{3}x^3$

b) $\int x^2\cdot 2^x dx = x^2\cdot\frac{2^x}{\ln 2}-\frac{2}{\ln 2}\cdot\int x\cdot 2^x dx = x^2\cdot\frac{2^x}{\ln 2}-\frac{2}{\ln 2}\left[x\frac{2^x}{\ln 2}-\int\frac{2^x}{\ln 2}dx\right]=\frac{2^x}{\ln 2}x^2-\frac{2^{x+1}}{(\ln 2)^2}x+\frac{2^{x+1}}{(\ln 2)^3}+k$

$u = x^2 \rightarrow du = 2xdx$

$dv = 2^x dx \rightarrow v = \frac{2^x}{\ }$

$u = x \rightarrow du = dx$

$dv = 2^x dx \rightarrow v = \frac{2^x}{\ }$

19. Página 275

a) $\int \frac{2}{x^2-1}dx = \int \left(\frac{-1}{x+1}+\frac{1}{x-1}\right)dx = \int \frac{-1}{x+1}dx + \int \frac{1}{x-1}dx = -\ln|x+1| + \ln|x-1| + k$

b) $\int -\frac{3}{x^2+x-2}dx = \int \frac{1}{x+2}dx + \int \frac{-1}{x-1}dx = \ln|x+2| - \ln|x-1| + k$

20. Página 275

a) $\int \frac{2x+1}{x^4-5x^2+4}dx = \int \left(\frac{\frac{5}{12}}{x-2}+\frac{\frac{-1}{2}}{x-1}+\frac{\frac{-1}{6}}{x+1}+\frac{\frac{1}{4}}{x+2}\right)dx = \frac{5}{12}\int \frac{1}{x-2}dx - \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{6}\int \frac{1}{x+1}dx + \frac{1}{4}\int \frac{1}{x+2}dx =$

$= \frac{5}{12}\ln|x-2| - \frac{1}{2}\ln|x-1| - \frac{1}{6}\ln|x+1| + \frac{1}{4}\ln|x+2| + k$

b) $\int \frac{7x-2}{x^3-2x^2-x+2}dx = \int \left(\frac{4}{x-2}+\frac{\frac{-5}{2}}{x-1}+\frac{\frac{-3}{2}}{x+1}\right)dx = 4\int \frac{1}{x-2}dx - \frac{5}{2}\int \frac{1}{x-1}dx - \frac{3}{2}\int \frac{1}{x+1}dx =$

$= 4\ln|x-2| - \frac{5}{2}\ln|x-1| - \frac{3}{2}\ln|x+1| + k$

21. Página 276

a) $\int \frac{x^2}{(x-1)^3}dx = \int \left(\frac{1}{(x-1)^3}+\frac{2}{(x-1)^2}+\frac{1}{x-1}\right)dx = \int \frac{1}{(x-1)^3}dx + \int \frac{2}{(x-1)^2}dx + \int \frac{1}{x-1}dx =$

$= \frac{-1}{2(x-1)^2} - \frac{2}{x-1} + \ln|x-1| + k$

b) $\int -\frac{3x-2}{(2-x)^2}dx = \int \left(\frac{-4}{(2-x)^2}+\frac{3}{2-x}\right)dx = 4\int \frac{-1}{(2-x)^2}dx - 3\int \frac{-1}{2-x}dx = \frac{-4}{2-x} - 3\ln|2-x| + k$

22. Página 276

a) $\int \frac{-2x^2+1}{x^3+6x^2+12x+8}dx = \int \left(\frac{-7}{(x+2)^3}+\frac{8}{(x+2)^2}+\frac{-2}{x+2}\right)dx =$

$= -7\int \frac{1}{(x+2)^3}dx + 8\int \frac{1}{(x+2)^2}dx - 2\int \frac{1}{x+2}dx = \frac{7}{2(x+2)^2} - \frac{8}{x+2} - 2\ln|x+2| + k$

b) $\int \frac{x-2}{x^4}dx = \int \left(\frac{1}{x^3}+\frac{-2}{x^4}\right)dx = \int \frac{1}{x^3}dx - 2\int \frac{1}{x^4}dx = \frac{-1}{2x^2} + \frac{2}{3x^3} + k$

23. Página 277

$\int \frac{4x^2-2x}{(x+2)(x-3)^2}dx = \int \left(\frac{6}{(x-3)^2}+\frac{\frac{16}{5}}{x-3}+\frac{\frac{4}{5}}{x+2}\right)dx =$

$= 6\int \frac{1}{(x-3)^2}dx + \frac{16}{5}\int \frac{1}{x-3}dx + \frac{4}{5}\int \frac{1}{x+2}dx = -\frac{6}{x-3} + \frac{16}{5}\ln|x-3| + \frac{4}{5}\ln|x+2| + k$

24. Página 277

$$\int \frac{-x^2+7x}{x^3-x^2-x+1}dx = \int\left(\frac{3}{(x-1)^2}+\frac{1}{x-1}+\frac{-2}{x+1}\right)dx = 3\int\frac{1}{(x-1)^2}dx+\int\frac{1}{x-1}dx-2\int\frac{1}{x+1}dx=$$

$$=-\frac{3}{x-1}+\ln|x-1|-2\ln|x+1|+k$$

25. Página 278

a) $\int \frac{2}{x^2+1}dx = 2arctgx + k$

b) $\int -\frac{3x-2}{2+x^2}dx = \int -\frac{3x}{2+x^2}dx+\int\frac{2}{2+x^2}dx = -\frac{3}{2}\ln|2+x^2|+\frac{2}{\sqrt{2}}arc\,tg\left(\frac{x}{\sqrt{2}}\right)+k=$

$$=-\frac{3}{2}\ln|2+x^2|+\sqrt{2}arc\,tg\left(\frac{x}{\sqrt{2}}\right)+k$$

26. Página 278

a) $\int \frac{-2x^2+1}{x^3-x^2+3x-3}dx = \int\left(\frac{\frac{-1}{4}}{x-1}+\frac{\frac{-7}{4}x-\frac{7}{4}}{x^2+3}\right)dx = \int\frac{\frac{-1}{4}}{x-1}dx+\int\frac{\frac{-7}{4}x-\frac{7}{4}}{x^2+3}dx=$

$$=\int\frac{\frac{-1}{4}}{x-1}dx+\int\frac{\frac{-7}{4}x}{x^2+3}dx+\int\frac{-\frac{7}{4}}{x^2+3}dx=-\frac{1}{4}\ln|x-1|-\frac{7}{8}\ln|3+x^2|-\frac{7}{4\sqrt{3}}arctg\left(\frac{x}{\sqrt{3}}\right)+k$$

b) $\int\frac{x-2}{x^2(x^2+1)}dx = \int\left(\frac{-2}{x^2}+\frac{1}{x}+\frac{-x+2}{x^2+1}\right)dx = \int\frac{-2}{x^2}dx+\int\frac{1}{x}dx+\int\frac{-x+2}{x^2+1}dx=$

$$=\int\frac{-2}{x^2}dx+\int\frac{1}{x}dx+\int\frac{-x}{x^2+1}dx+\int\frac{2}{x^2+1}dx=\frac{2}{x}+\ln|x|-\frac{1}{2}\ln|1+x^2|+2arc\,tg\,x+k$$

27. Página 279

a) $\int\frac{2x^4}{(x-1)^3}dx = \int\left(2x+6+\frac{12}{x-1}+\frac{8}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx = x^2+6x+12\ln|x-1|-\frac{8}{x-1}-\frac{1}{(x-1)^2}+k$

b) $\int -\frac{3x^3-2}{(2-x)^2}dx = \int\left(-3x-12+\frac{36}{2-x}-\frac{22}{(2-x)^2}\right)dx = \frac{-3x^2}{2}-12x-36\ln|2-x|-\frac{22}{2-x}+k$

28. Página 279

a) $\int\frac{-2x^5+1}{x^4-2x^2+1}dx = \int\left(-2x+\frac{\frac{-1}{4}}{(x-1)^2}+\frac{\frac{-9}{4}}{x-1}+\frac{\frac{3}{4}}{(x+1)^2}+\frac{\frac{-7}{4}}{x+1}\right)dx=$

$$=\int -2xdx+\int\frac{\frac{-1}{4}}{(x-1)^2}dx+\int\frac{\frac{-9}{4}}{x-1}dx+\int\frac{\frac{3}{4}}{(x+1)^2}dx+\int\frac{\frac{-7}{4}}{x+1}dx=$$

$$=-x^2+\frac{1}{4(x-1)}-\frac{9}{4}\ln|x-1|-\frac{3}{4(x+1)}-\frac{7}{4}\ln|x+1|+k$$

b) $\int\frac{x^6-1}{x^2(x^2+1)(x-1)}dx = \int\left(x+1+\frac{1}{x^2}+\frac{1}{x}+\frac{-x}{x^2+1}+\frac{-1}{x^2+1}\right)dx = \frac{x^2}{2}+x-\frac{1}{x}+\ln|x|-\frac{1}{2}\ln|x^2+1|-arc\,tg\,x+k$

29. Página 280

a) $\int x\cdot 2^{x^2-3}dx = \frac{1}{2}\int 2^t dt = \frac{1}{2}\left(\frac{2^t}{\ln 2}\right)+k = \frac{2^{x^2-3}}{2\ln 2}+k = \frac{2^{x^2-4}}{\ln 2}+k$

$t = x^2-3 \rightarrow dt = 2xdx \rightarrow \frac{dt}{2} =$

b) $\int \frac{\ln^3 x}{2x}dx = \int \frac{t^3}{2}dt = \frac{1}{2}\frac{t^4}{4}+k = \frac{\ln^4 x}{8}+k$

$t = \ln x \rightarrow dt = \frac{1}{x}dx$

$dv = \frac{1}{2}dt \rightarrow v = \frac{1}{2}t$

c) $\int x\cdot\ln(1+x^2)dx = \int \frac{1}{2}\ln t\,dt = \frac{1}{2}t\ln t - \int \frac{1}{2}t\cdot\frac{1}{t}dt = \frac{t}{2}\ln t - \frac{t}{2}+k = \frac{1+x^2}{2}\ln(1+x^2) - \frac{1+x^2}{2}+k$

$t = 1+x^2 \rightarrow dt = 2xdx \rightarrow \frac{dt}{2} =$

d) $\int \frac{1}{e^x+e^{-x}}dx = \int \frac{1}{t+\frac{1}{t}}\cdot\frac{1}{t}dt = \int\frac{1}{t^2+1}dt = arctg(t)+k = arctg(e^x)+k$

$t = e^x \rightarrow dt = e^x dx \rightarrow \frac{dt}{t} =$

30. Página 280

a) $\int \frac{x^2+2}{\sqrt{x^3+6x}}dx = \int \frac{1}{3\sqrt{t}}dt = \frac{1}{3}\cdot 2t^{\frac{1}{2}}+k = \frac{2}{3}\sqrt{x^3+6x}+k$

$t = x^3+6x \rightarrow dt = (3x^2+6)dx \rightarrow \frac{dt}{3} =$

b) $\int\frac{x}{1+x^4}dx = \frac{1}{2}\int\frac{1}{1+t^2}dt = \frac{1}{2}arctg(t)+k = \frac{1}{2}arctg(x^2)+k$

$t = x^2 \rightarrow dt = 2xdx \rightarrow \frac{dt}{2} =$

c) $\int\frac{arctg\,x}{1+x^2}dx = \int t\,dt = \frac{t^2}{2}+k = \frac{(arctg\,x)^2}{2}+k$

$t = arctg\,x \rightarrow dt = \frac{1}{\ldots}$

d) $\int\frac{dx}{(arc\,sen\,x)^5\sqrt{1-x^2}} = \int\frac{dt}{t^5} = -\frac{1}{4t^4}+k = -\frac{1}{4(arc\,sen\,x)^4}+k$

$t = arc\,sen\,x \rightarrow dt = \frac{1}{\ldots}$

31. Página 281

a) $\int sen^5 x\cos^2 x dx = \int senx(1-\cos^2 x)^2\cos^2 x dx = -\int (1-t^2)^2 t^2 dt = -\int (t^6 - 2t^4 + t^2)dt =$

$t = \cos x \rightarrow dt = -sen\, x\, dx$

$= -\frac{t^7}{7} + \frac{2t^5}{5} - \frac{t^3}{3} + k = -\frac{\cos^7 x}{7} + \frac{2\cos^5 x}{5} - \frac{\cos^3 x}{3} + k$

b) $\int \sqrt{4-x^2}dx = 2\int \sqrt{1-\left(\frac{x}{2}\right)^2}dx = 2\int \sqrt{1-sen^2 t}\cdot 2\cos t\, dt = 4\int \cos^2 t\, dt = 4\int \left(\frac{1+\cos 2t}{2}\right)dt$

$sent = \frac{x}{2} \rightarrow dx = 2\cos t\, dt$

$= 2t + 2sen2t + k = 2arc\,sen\frac{x}{2} + 2sen2\left(arc\,sen\frac{x}{2}\right) + k = 2arc\,sen\frac{x}{2} + 2\cdot\frac{x}{2}\sqrt{1-\left(\frac{x}{2}\right)^2} + k =$

$= 2arc\,sen\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2} + k$

32. Página 281

a) $\int sen^2 x dx = \int \frac{1-\cos 2x}{2}dx = \frac{x}{2} - \frac{sen 2x}{4} + k$

b) $\int \frac{\sqrt{2-x^2}}{4}dx = \frac{\sqrt{2}}{4}\int \sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2}dx = \frac{\sqrt{2}}{4}\int \sqrt{1-sen^2 t}\sqrt{2}\cos t\, dt =$

$sent = \frac{x}{\sqrt{2}} \rightarrow dx = \sqrt{2}\cos t\, dt$

$= \frac{1}{2}\int \cos^2 t dt = \frac{1}{2}\int \frac{1+\cos 2t}{2}dt = \frac{1}{4}\left(t + \frac{sen 2t}{2}\right) + k = \frac{1}{4}arc\,sen\frac{x}{\sqrt{2}} + \frac{x}{8}\sqrt{2-x^2} + k$

SABER HACER

33. Página 282

$f(x) = \int \frac{2x}{x^2+1}dx = \ln|x^2+1| + k$

$f(0) = 1 \rightarrow \ln|1| + k = 1 \rightarrow k = 1 \rightarrow f(x) = \ln|x^2+1| + 1$

34. Página 282

$F(x) = \int \frac{x^2}{\sqrt{1-x^3}}dx = -\frac{2}{3}\sqrt{1-x^3} + k$

$F(0) = 0 \rightarrow -\frac{2}{3} + k = 0 \rightarrow k = \frac{2}{3} \rightarrow F(x) = -\frac{2}{3}\sqrt{1-x^3} + \frac{2}{3}$

35. Página 282

$\int \frac{dx}{\sqrt{4-(2x+1)^2}} = \int \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{4-(2x+1)^2}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{2x+1}{2}\right)^2}}dx = \frac{1}{2}arc\,sen\left(\frac{2x+1}{2}\right) + k$

36. Página 282

$$\int e^{2x+e^{2x}}dx = \int t\cdot e^t\cdot\frac{1}{2t}dt = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + k = \frac{1}{2}e^{e^{2x}} + k$$

$$t = e^{2x} \rightarrow dt = 2e^{2x}dx \rightarrow dx = \frac{dt}{2t}$$

37. Página 283

$$x = 5 sen\, t \rightarrow dx = 5\cos t\, dt$$

$$\int\frac{\sqrt{25-x^2}}{5}dx = \int\frac{\sqrt{25-25 sen^2 t}}{5}\cdot 5\cos t\, dt = \int\frac{\sqrt{25(1-sen^2 t)}}{5}\cdot 5\cos t\, dt = 5\int\sqrt{1-sen^2 t}\,\cos t\, dt =$$

$$= 5\int\sqrt{\cos^2 t}\,\cos t\, dt = 5\int\cos^2 t\, dt = 5\int\frac{1+\cos 2t}{2}dt = \frac{5}{2}\int dt + 5\int\frac{\cos 2t}{2}dt = \frac{5}{2}t + \frac{5}{4}sen\, 2t + k$$

$$x = 5 sen\, t \rightarrow t = arc\,sen\left(\frac{x}{5}\right)$$

$$sen\, 2t = 2 sen\, t\cos t = 2\left(\frac{x}{5}\right)\sqrt{1-\left(\frac{x}{5}\right)^2} = \frac{2x}{\ldots}\sqrt{25-x^2} = \ldots$$

$$\int\frac{\sqrt{25-x^2}}{5}dx = \frac{5}{2}t + \frac{5}{4}sen\, 2t + k = \frac{5}{2}arc\,sen\left(\frac{x}{5}\right) + \frac{5}{4}\cdot\frac{2x}{25}\sqrt{25-x^2} + k = \frac{5}{2}arc\,sen\left(\frac{x}{5}\right) + \frac{x}{10}\sqrt{25-x^2} + k$$

38. Página 283

$$I = \int e^x sen\, x\, dx = -e^x\cos x + \int\cos x\, e^x dx = -e^x\cos x + e^x sen\, x - \int e^x sen\, x\, dx = -e^x\cos x + e^x sen\, x - I$$

$$u = e^x \rightarrow du = e^x dx$$
$$dv = sen\, x\, dx \rightarrow v = -c$$

Despejamos I:

$$I = -e^x\cos x + e^x sen\, x - I \;\rightarrow\; I = \frac{e^x(sen\, x - \cos x)}{2}$$

Entonces: $\displaystyle\int e^x sen\, x\, dx = \frac{e^x(sen\, x-\cos x)}{2} + k$

39. Página 284

$$\int 3\ln(2x-1)dx = 3x\ln(2x-1) - \int\frac{6x}{2x-1}dx = 3x\ln(2x-1) - \int\left(3+\frac{3}{2x-1}\right)dx = 3x\ln(2x-1) - 3x - \frac{3}{2}\ln|2x-1| + k$$

$$u = \ln(2x-1) \rightarrow du = \frac{2}{2x-}$$
$$dv = 3dx \rightarrow v = 3x$$

$$\frac{6x}{2x-1} = 3 + \frac{3}{2x-1}$$

40. Página 284

$$\int(x^2+1)\cdot e^{x-2}dx = (x^2+1)\cdot e^{x-2} - \int 2xe^{x-2}dx = (x^2+1)\cdot e^{x-2} - 2xe^{x-2} + \int 2e^{x-2}dx =$$

$$u = x^2+1 \rightarrow du = 2xd$$
$$dv = e^{x-2}dx \rightarrow v = e^{x}$$

$$u = 2x \rightarrow du = 2dx$$
$$dv = e^{x-2}dx \rightarrow v = e^{x-}$$

$= (x \ldots e^{x-2} + k = e \ldots (x \ldots 2x + 3) + k$

41. Página 285

$$\int \frac{\sqrt{x+1}+2}{(x+1)^{2/3}-\sqrt{x+1}}dx = \int \frac{\sqrt{t^6}+2}{(t^6)^{2/3}-\sqrt{t^6}}\cdot 6t^5 dt = 6\int \frac{t^3+2}{t^4-t^3}\cdot t^5 dt =$$

$$t^6 = 1+x \rightarrow 6t^5dt = dx \rightarrow t = (1+x)^{1/6}$$

$$= 6\int \frac{t^5+2t^2}{t-1}dt = 6\int\left(t^4+t^3+t^2+3t+3+\frac{3}{t-1}\right)dt = \frac{6}{5}t^5+\frac{3}{2}t^4+2t^3+9t^2+18t+18\ln|t-1|+k =$$

$$= \frac{6}{5}(1+x)^{5/6}+\frac{3}{2}(1+x)^{2/3}+2(1+x)^{1/2}+9(1+x)^{1/3}+18(1+x)^{1/6}+18\ln\left|(1+x)^{1/6}-1\right|+k$$

42. Página 285

$$\int \frac{1+x}{1+\sqrt{x}}dx = \int \frac{1+t^2}{1+t}\cdot 2tdt = 2\int \frac{t^3+t}{1+t}dt = 2\int\left(t^2-t+2-\frac{2}{t+1}\right)dt = \frac{2}{3}t^3-t^2+4t-4\ln|t+1|+k =$$

$$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}dx \rightarrow dx = 2tdt$$

$$= \frac{2}{3}\sqrt{x^3}-x+4\sqrt{x}-4\ln\left|\sqrt{x}+1\right|+k$$

ACTIVIDADES FINALES

43. Página 286

a) $F(x) = 3x^5-2x+1 \rightarrow F'(x) = 15x^4-2 \rightarrow F(x)$ es primitiva de *f*(*x*).

b) $F(x) = (x^5-2)^3 \rightarrow F'(x) = 15x^4(x^5-2)^2 \rightarrow F(x)$ es primitiva de *f*(*x*).

c) $F(x) = \frac{2-x}{x^3}+7 \rightarrow F'(x) = \frac{2x-6}{x^4} \rightarrow F(x)$ es primitiva de *f*(*x*).

d) $F(x) = \frac{x-1}{x^2}-3 \rightarrow F'(x) = \frac{-x+2}{x^3} \rightarrow F(x)$ es primitiva de *f*(*x*).

e) $F(x) = \ln\sqrt{x^2+2x} \rightarrow F'(x) = \frac{x+1}{x^2+2x} \rightarrow F(x)$ es primitiva de *f*(*x*).

f) $F(x) = 5e^{-x^2}+11 \rightarrow F'(x) = -10xe^{-x^2} \rightarrow F(x)$ es primitiva de *f*(*x*).

g) $F(x) = arc\,tg\sqrt{x} \rightarrow F'(x) = \frac{1}{2\sqrt{x}(1+x)} \rightarrow F(x)$ es primitiva de *f*(*x*).

h) $F(x) = cos^2x-1492 \rightarrow F'(x) = 2sen\,x\,cos\,x = -sen\,2x \rightarrow F(x)$ es primitiva de *f*(*x*).

44. Página 286

a) $f(x) = 3x^2 \rightarrow F(x) = x^3+k$ $\qquad F(0) = 1 \rightarrow k = 1 \rightarrow F(x) = x^3+1$

b) $f(x) = \frac{4}{5x} \rightarrow F(x) = \frac{4}{5}\ln|x|+k$ $\qquad F(1) = 4 \rightarrow k = 4 \rightarrow F(x) = \frac{4}{5}\ln|x|+4$

c) $f(x) = sen\,3x \rightarrow F(x) = -\frac{1}{3}cos\,3x+k$ $\qquad F(\pi) = -\frac{1}{3} \rightarrow k = -\frac{2}{3} \rightarrow F(x) = -\frac{1}{3}cos\,3x-\frac{2}{3}$

d) $f(x) = e^{2x} \rightarrow F(x) = \frac{e^{2x}}{2}+k$ $\qquad F(0) = \frac{2}{3} \rightarrow k = \frac{1}{6} \rightarrow F(x) = \frac{e^{2x}}{2}+\frac{1}{6}$

e) $f(x) = \frac{3}{4}(x^2-1) \rightarrow F(x) = \frac{x^3}{4}-\frac{3x}{4}+k$ $\qquad F(1) = \frac{1}{4} \rightarrow k = \frac{1}{4}+\frac{3}{4}-\frac{1}{4} = \frac{3}{4}$

45. Página 286

a) $F(x)=1001x+k\rightarrow F'(x)=1001$

b) $F(x)=x^2+k\rightarrow F'(x)=2x$

c) $F(x)=\dfrac{x^2}{2}+k\rightarrow F'(x)=x$

d) $F(x)=x^3+k\rightarrow F'(x)=3x^2$

e) $F(x)=\dfrac{x^3}{3}+k\rightarrow F'(x)=x^2$

f) $F(x)=x^4+k\rightarrow F'(x)=4x^3$

g) $F(x)=x^{n+1}+k\rightarrow F'(x)=(n+1)x^n$

h) $F(x)=\dfrac{x^{n+1}}{n+1}+k\rightarrow F'(x)=x^n$

46. Página 286

a) $F(x)=\sqrt{x}+k\rightarrow F'(x)=\dfrac{1}{2\sqrt{x}}$

b) $F(x)=2\sqrt{x}+k\rightarrow F'(x)=\dfrac{1}{\sqrt{x}}$

c) $F(x)=2\sqrt{x+1}+k\rightarrow F'(x)=\dfrac{1}{\sqrt{x+1}}$

d) $F(x)=2\sqrt{x-4}+k\rightarrow F'(x)=\dfrac{1}{\sqrt{x-4}}$

e) $F(x)=\dfrac{2}{3}\sqrt{3x+10}+k\rightarrow F'(x)=\dfrac{1}{\sqrt{3x+10}}$

f) $F(x)=arc\,sen\,x+k\rightarrow F'(x)=\dfrac{1}{\sqrt{1-x^2}}$

47. Página 286

a) $F(x)=19\ln|x|+k\rightarrow F'(x)=\dfrac{19}{x}$

b) $F(x)=\dfrac{\ln|x|}{19}+k\rightarrow F'(x)=\dfrac{1}{19x}$

c) $F(x)=\ln|19+x|+k\rightarrow F'(x)=\dfrac{1}{19+x}$

d) $F(x)=2\ln|2x+3|+k\rightarrow F'(x)=\dfrac{4}{2x+3}$

e) $F(x)=\dfrac{1}{x}+k\rightarrow F'(x)=-\dfrac{1}{x^2}$

f) $F(x)=-arc\,tg\,x+k\rightarrow F'(x)=-\dfrac{1}{1+x^2}$

48. Página 286

a) $F(x)=e^x+k\rightarrow F'(x)=e^x$

b) $F(x)=\dfrac{e^{2x}}{2}+k\rightarrow F'(x)=e^{2x}$

c) $F(x)=\dfrac{e^{5x+55}}{5}+k\rightarrow F'(x)=e^{5x+55}$

d) $F(x)=\dfrac{2^x}{\ln 2}+k\rightarrow F'(x)=2^x$

e) $F(x)=\dfrac{2^{x-7}}{\ln 2}+k\rightarrow F'(x)=2^{x-7}$

f) $F(x)=\dfrac{2^{9x+5}}{9\ln 2}+k\rightarrow F'(x)=2^{9x+5}$

49. Página 286

a) $F(x)=sen\,x+k\rightarrow F'(x)=cos\,x$

b) $F(x)=\dfrac{sen\,3x}{3}+k\rightarrow F'(x)=cos\,3x$

c) $F(x)=sen(x+3)+k\rightarrow F'(x)=cos(x+3)$

d) $F(x)=-3cos\,x+k\rightarrow F'(x)=3sen\,x$

e) $F(x)=-cos(x-\pi)+k\rightarrow F'(x)=sen(x-\pi)$

f) $F(x)=-3cos(x-\pi)+k\rightarrow F'(x)=3sen(x-\pi)$

50. Página 286

a) Porque la derivada de una función polinómica siempre es otra función polinómica.

b) El grado de $F(x)$ es $n+1$, considerando que $f(x)$ es un polinomio o que n sea distinto de -1.

51. Página 286

a) $\int (2x-3)dx = x^2 - 3x + k$

b) $\int (3x^2 + 4x - 2)dx = x^3 + 2x^2 - 2x + k$

c) $\int \left(\frac{3}{4}x^3 - 3x^2 + 6x - 1\right)dx = \frac{3}{16}x^4 - x^3 + 3x^2 - x + k$

d) $\int (x-1)^3 dx = \frac{(x-1)^3}{4} + k$

e) $\int (x+3)^2 dx = \frac{(x+3)^3}{3} + k$

f) $\int (1-2x)^2 dx = \frac{-1}{2}\int (-2)\cdot(1-2x)^2 dx = \frac{-1}{2}\frac{(1-2x)^3}{3} + k = \frac{-1}{6}(1-2x)^3 + k$

g) $\int \left(\frac{x^3}{3} - \frac{x^2}{2} + x - 5\right)dx = \frac{x^4}{12} - \frac{x^3}{6} + \frac{x^2}{2} - 5x + k$

h) $\int (x - 3x + 2)dx = \int (-2x + 2)dx = -x^2 + 2x + k$

52. Página 286

a) $\int 2\sqrt{x}dx = \frac{4}{3}\sqrt{x^3} + k$

b) $\int 2\sqrt{3x}\,dx = 2\sqrt{3}\int x^{\frac{1}{2}}dx = \frac{4\sqrt{3}}{3}x^{\frac{3}{2}} + k = \frac{4}{3}\sqrt{3x^3} + k$

c) $\int \left(\frac{1}{\sqrt{x}} - x\right)dx = 2\sqrt{x} - \frac{x^2}{2} + k$

d) $\int \left(\sqrt[4]{x} + \sqrt[3]{x}\right)dx = \frac{4}{5}\sqrt[4]{x^5} + \frac{3}{4}\sqrt[3]{x^4} + k$

e) $\int (2 + \sqrt{x})dx = 2x + \frac{2}{3}\sqrt{x^3} + k$

f) $\int (2 + \sqrt{3x})dx = 2x + \frac{2}{3}\sqrt{3x^3} + k$

g) $\int \left(\frac{1}{\sqrt[3]{x}} - x\right)dx = \frac{3}{2}\sqrt[3]{x^2} - \frac{x^2}{2} + k$

h) $\int \left(\frac{2}{\sqrt{2x}} + \frac{4}{\sqrt[4]{4x}}\right)dx = \sqrt{2}\int x^{-\frac{1}{2}}dx + 2\sqrt{2}\int x^{-\frac{1}{4}}dx = 2\sqrt{2}x^{\frac{1}{2}} + \frac{8\sqrt{2}}{3}x^{\frac{3}{4}} + k = 2\sqrt{2x} + \frac{8\sqrt{2}}{3}\sqrt[4]{x^3} + k$

53. Página 286

a) $\int \frac{4}{x-2}dx = 4\ln|x+2| + k$

b) $\int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}\right)dx = \ln|x| - \frac{2}{x} - \frac{3}{2x^2} + k$

c) $\int \left(\frac{2}{x+3} - \frac{1}{x-4}\right)dx = 2\ln|x+3| - \ln|x-4| + k$

d) $\int \frac{1}{(x+4)^2}dx = -\frac{1}{x-4} + k$

e) $\int \left(\frac{1}{(x-1)^2} + \frac{7}{x+3}\right)dx = \frac{-1}{x-1} + 7\ln|x+3| + k$

f) $\int \left(\frac{2}{(x+3)^3} + \frac{5}{(x-3)^2}\right)dx = \frac{-1}{(x+3)^2} - \frac{5}{(x-3)} + k$

54. Página 287

a) $\int e^{x-2}\,dx = e^{x-2}+k$

b) $\int (e^x+1)dx = e^x + x + k$

c) $\int (e^{2x}+2^x)dx = \frac{1}{2}e^{2x}+\frac{2^x}{\ln 2}+k$

d) $\int \left(xe^{x^2}+\frac{4}{3}x^3\right)dx = \frac{1}{2}e^{x^2}+\frac{1}{3}x^4+k$

e) $\int e^{2x+3}\,dx = \frac{1}{2}e^{2x+3}+k$

f) $\int (2e^x-3x^2)dx = 2e^x - x^3 + k$

g) $\int \left(5e^{\frac{x}{2}}+2\cdot 3^x\right)dx = 10e^{\frac{x}{2}}+2\frac{3^x}{\ln 3}+k$

h) $\int (x2^{x^2+2x}+2^{x^2+2x})dx = \frac{1}{2}\int (2x+2)\cdot 2^{x^2+2x}\,dx = \frac{1}{2}\cdot\frac{2^{x^2+2x}}{\ln 2}+k$

55. Página 287

a) $\int cos(2x)dx = \frac{1}{2}sen(2x)+k$

b) $\int 4sen(x+\pi)dx = -4cos(x+\pi)+k$

c) $\int 3cos\left(\frac{\pi}{2}-\frac{x}{3}\right)dx = -9sen\left(\frac{\pi}{2}-\frac{x}{3}\right)+k$

d) $\int 5sen(2x-\pi)dx = \frac{-5}{2}cos(2x-\pi)+k$

e) $\int 3sec^2\left(\frac{1}{5}x\right)dx = 15\int \frac{\frac{1}{5}dx}{cos^2\left(\frac{1}{5}x\right)} = 15tg\left(\frac{1}{5}x\right)+k$

f) $\int \frac{7}{sen^2(3x)}dx = \frac{7}{3}\int \frac{3}{sen^2(3x)}dx = -\frac{7}{3}cotg(3x)+k$

g) $\int \frac{5}{cos^2\left(\frac{x}{3}\right)}dx = 15\int \frac{\left(\frac{1}{3}\right)}{cos^2\left(\frac{x}{3}\right)}dx = 15tg\left(\frac{x}{3}\right)+k$

h) $\int \frac{3}{\sqrt{1-x^2}}dx = 3arc\,sen\,x+k$

i) $\int \frac{3}{x^2+1}dx = 3arc\,tg\,x+k$

j) $\int \frac{1}{(3x)^2+1}dx = \frac{1}{3}arc\,tg(3x)+k$

56. Página 287

a) Consideramos $\int x^2\,dx$ y comprobamos que no coincide con el producto $\int x\,dx\cdot\int x\,dx$.

$$\int x^2dx = \frac{x^3}{3}+k$$

$$\int xdx = \frac{x^2}{2} \rightarrow \int xdx\cdot\int xdx = \frac{x^4}{4}+k$$

Entonces la afirmación es cierta.

b) Consideramos $\int 1dx$ y comprobamos que no coincide con $\dfrac{\int xdx}{\int xdx}$.

$$\int 1dx = \int \frac{x}{x}dx = x+k$$

$$\int xdx = \frac{x^2}{2} \rightarrow \frac{\int xdx}{\int xdx} = \frac{\frac{x^2}{2}}{\frac{x^2}{2}}+k = 1+k$$

Entonces la afirmación es cierta.

57. Página 287

a) $\int \frac{2x}{x^2+1}dx = \ln|x^2+1| + k$

b) $\int \frac{8x-3}{4x^2-3x+1}dx = \ln|4x^2-3x+1| + k$

c) $\int \frac{6x^2+1}{2x^3+x-9}dx = \ln|2x^3+x-9| + k$

d) $\int \frac{senx}{cosx}dx = -\ln|cosx| + k$

e) $\int cotgdx = \int \frac{cosx}{senx}dx = \ln|senx| + k$

f) $\int 3x^2\, sen\, x^3\, dx = -cos\, x^3 + k$

g) $\int (2x+1)sen(x^2+x+5)dx = -cos(x^2+x+5) + k$

h) $\int 6xcos(3x^2-5)dx = sen(3x^2-5) + k$

i) $\int \left(\frac{1}{x^2+1} + \frac{1}{x^2}\right)dx = arc\,tg\, x - \frac{1}{x} + k$

j) $\int 6xe^{3x^2}dx = e^{3x^2} + k$

k) $\int (3x^2+1)e^{x^3+x}dx = e^{x^3+x} + k$

l) $\int (12x^2-6x)e^{4x^3-3x^2+7}dx = e^{4x^3-3x^2+7} + k$

m) $\int xe^{7x^2}dx = \frac{e^{7x^2}}{14} + k$

n) $\int \frac{-2}{4+x^2}dx = \frac{-1}{2}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx = -arc\,tg\frac{x}{2} + k$

ñ) $\int \frac{-2}{\sqrt{3-x^2}}dx = \frac{-2}{\sqrt{3}}\int \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{3}}\right)^2}}dx = -2arc\,sen\frac{x}{\sqrt{3}} + k$

o) $\int x^3e^{x^2} = \frac{1}{2}\int te^t dt = \frac{1}{2}e^{x^2}(x^2-1) + k$ (con $t = x^2$ y $dt = 2xdx$)

p) $\int \frac{x+\ln x}{x}dx = \int dx + \int \frac{\ln x}{x}dx = x + \int \frac{\ln x}{x}dx = x + \frac{\ln^2 x}{2} + k$

q) $\int \left(\frac{1}{x^2+1} - \frac{2}{x^2} + \frac{3}{x}\right)dx = arc\,tg\,x + \frac{2}{x} + 3\ln|x| + k$

58. Página 287

a) $\int \frac{x}{x^2-3}dx = \frac{1}{2}\int \frac{2x}{x^2-3}dx = \frac{1}{2}\ln\left|x^2-3\right| + k$

b) $\int \frac{x^3}{\sqrt{x^4+3}}dx = \frac{1}{4}\int \frac{1}{\sqrt{t}}dt = \frac{1}{2}\sqrt{t} + k = \frac{1}{2}\sqrt{x^4+3} + k$

$t = x^4+3 \rightarrow dt = 4x^3\,dx \rightarrow x^3\,dx = \frac{1}{4}dt$

c) $\int \frac{sen\,x}{1-cos\,x}dx = \ln\left|1-cos\,x\right| + k$

d) $\int \frac{3+x}{\sqrt{1-x^2}}dx = \int \frac{3}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx =$

$t = 1-x^2 \rightarrow dt = -2x\,dx \rightarrow x\,dx = \frac{1}{2}dt$

$= 3arc\,senx - \frac{1}{2}\int \frac{1}{\sqrt{t}}dt = 3arc\,senx - \sqrt{1-x^2} + k$

e) $\int \frac{2x+\sqrt{x}}{x^2}dx = \int \frac{2x}{x^2}dx + \int \frac{\sqrt{x}}{x^2}dx =$

$= \int \frac{2x}{x^2}dx + \int x^{-\frac{3}{2}}dx = \ln\left|x^2\right| - \frac{2}{\sqrt{x}} + k$

f) $\int \frac{2cos\,x}{3+sen\,x}dx = 2\ln\left|3+sen\,x\right| + k$

g) $\int \frac{x}{x^2+2^4}dx = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|x^2+2^4| + k$

$t = x^2+2^4 \rightarrow dt = 2x\,dx$

h) $\int \frac{x}{\sqrt{1+3x^2}}dx = \frac{1}{6}\int \frac{1}{\sqrt{t}}dt = \frac{2}{6}\sqrt{t} + k = \frac{1}{3}\sqrt{1+3x^2} + k$

$t = 1+3x^2 \rightarrow dt = 6x\,dx \rightarrow x\,dx = \frac{1}{6}dt$

i) $\int \frac{x-1}{x^2+1}dx = \int \frac{x}{x^2+1}dx + \int \frac{-1}{x^2+1}dx =$

$= \frac{1}{2}\ln\left|x^2+1\right| - arc\,tg\,x + k$

j) $\int \frac{8}{x^2+4}dx = \int \frac{2}{\left(\frac{x}{2}\right)^2+1}dx = 4\int \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}dx =$

$= 4arc\,tg\left(\frac{x}{2}\right) + k$

59. Página 287

$$F(x) = \int (x+1)(x^2+2x+6)\,dx = \int (x^3+3x^2+8x+6)\,dx = \frac{x^4}{4} + x^3 + 4x^2 + 6x + k$$

$$F(0) = 1 \rightarrow k = 1 \rightarrow F(x) = \frac{x^4}{4} + x^3 + 4x^2 + 6x + 1$$

60. Página 287

$$f(x) = \int \frac{a}{1+x}\,dx = a\ln|1+x| + k$$

$$f(0) = 1 \rightarrow a\ln 1 + k \rightarrow k = 1 \qquad f(1) = -1 \rightarrow a\ln 2 + 1 = -1 \rightarrow a = \frac{-2}{\ln 2}$$

La función es: $f(x) = \frac{-2}{\ln 2}\ln|1+x| + 1$

61. Página 287

$$F(x) = \int \frac{-x}{1-x^2}\,dx = \frac{1}{2}\int \frac{-2x}{1-x^2}\,dx = \frac{1}{2}\ln\left|1-x^2\right| + k$$

$$F(\sqrt{2}) = 3 \rightarrow \frac{1}{2}\ln|1-2| + k = 3 \rightarrow k = 3$$

La función primitiva es: $F(x) = \frac{1}{2}\ln\left|1-x^2\right| + 3$

62. Página 287

Sabemos que *f* pasa por el origen de coordenadas, por lo que: $f(0) = 0$

Además, ese es un punto de inflexión, entonces: $f''(0) = 0$

Como la pendiente de la recta tangente en (0, 0) es 5, podemos concluir que: $f'(0) = 5$

Finalmente, tenemos:

$$f'''(x) = 24x - 6$$

$$f''(x) = \int (24x-6)\,dx = 12x^2 - 6x + k \rightarrow f''(0) = k = 0 \rightarrow f''(x) = 12x^2 - 6x$$

$$f'(x) = \int (12x^2 - 6x)\,dx = 4x^3 - 3x^2 + k \rightarrow f'(0) = 5 \rightarrow k = 5 \rightarrow f'(x) = 4x^3 - 3x^2 + 5$$

$$f(x) = \int (4x^3 - 3x^2 + 5)\,dx = x^4 - x^3 + 5x + k \rightarrow f(0) = 0 \rightarrow k = 0 \rightarrow f(x) = x^4 - x^3 + 5x$$

63. Página 287

$$f''(x) = \int (x+1)\,dx = \frac{x^2}{2} + x + k \rightarrow f''(0) = 1 \rightarrow k = 1 \rightarrow f''(x) = \frac{x^2}{2} + x + 1$$

$$f'(x) = \int \left(\frac{x^2}{2} + x + 1\right)dx = \frac{x^3}{6} + \frac{x^2}{2} + x + k \rightarrow f'(0) = 5 \rightarrow k = 5 \rightarrow f'(x) = \frac{x^3}{6} + \frac{x^2}{2} + x + 5$$

$$f(x) = \int \left(\frac{x^3}{6} + \frac{x^2}{2} + x + 5\right)dx = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 5x + k \rightarrow f(0) = 0 \rightarrow k = 0 \rightarrow f(x) = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 5x$$

64. Página 287

$$F(x)=\int\frac{1-\operatorname{sen}x}{2x+2\cos x}dx=\frac{1}{2}\int\frac{1-\operatorname{sen}x}{x+\cos x}dx=\frac{1}{2}\ln|x+\cos x|+k$$

$$F(0)=2\rightarrow 0+k=2\rightarrow k=2\rightarrow F(x)=\frac{1}{2}\ln|x+\cos x|+2$$

65. Página 287

$$f'(x)=\int \operatorname{sen}x\,dx=-\cos x+k \qquad f(x)=\int(-\cos x+k)dx=-\operatorname{sen}x+kx+l$$

$$f(0)=1\rightarrow l=1\rightarrow f(x)=-\operatorname{sen}x+kx+1 \qquad f\left(\frac{\pi}{2}\right)=\pi\rightarrow -1+\frac{\pi}{2}k+1=\pi\rightarrow k=2\rightarrow f(x)=-\operatorname{sen}x+2x+1$$

66. Página 287

$$f'(x)=\int(6x+2)dx=3x^2+2x+k$$

f tiene un mínimo relativo en *A*(1, 3); por tanto, *f*(1) = 3 y *f*′(1) = 0.

$$f'(1)=0\rightarrow 3+2+k=0\rightarrow k=-5\rightarrow f'(x)=3x^2+2x-5$$

$$f(x)=\int(3x^2+2x-5)dx=x^3+x^2-5x+k\rightarrow f(1)=3\rightarrow 1+1-5+k=3\rightarrow k=6\ \rightarrow\ f(x)=x^3+x^2-5x+6$$

67. Página 288

a) $F(x)=\int\frac{3x^2+\cos x+2\cdot e^{2x}}{x^3+\operatorname{sen}x+e^{2x}}dx=\ln|x^3+\operatorname{sen}x+e^{2x}|+k\rightarrow F(0)=-5\rightarrow k=-5$

$F(x)=\ln|x^3+\operatorname{sen}x+e^{2x}|-5$

b) $F(0)=k$ Basta con tomar $k=0$. Entonces $F(x)=\ln|x^3+\operatorname{sen}x+e^{2x}|$ pasa por el origen de coordenadas.

c) $F(0)=k$ Basta con tomar $k=1$. Entonces $F(x)=\ln|x^3+\operatorname{sen}x+e^{2x}|+1$ pasa por el punto *B*(0, 1).

68. Página 288

a) Tenemos que *f*(–1) = –4. $\qquad f(x)=\begin{cases}f_1(x) & \text{si } x\le 1\\ f_2(x) & \text{si } x>1\end{cases}$

$$f_1(x)=\int(2-x)dx=2x-\frac{x^2}{2}+k\rightarrow f_1(-1)=-4\rightarrow -2-\frac{1}{2}+k=-4\rightarrow k=-\frac{3}{2}$$

$$f_1(x)=2x-\frac{x^2}{2}-\frac{3}{2}\ \text{si}\ x\le 1 \qquad f_2(x)=\int\frac{1}{x}dx=\ln|x|+k\ \text{si}\ x>1$$

Como *f* es derivable, entonces es continua; por tanto: $f_1(1)=f_2(1)\rightarrow 2-\frac{1}{2}-\frac{3}{2}=0=k\rightarrow f_2(x)=\ln|x|$

Entonces: $f(x)=\begin{cases}f_1(x)=2x-\frac{x^2}{2}-\frac{3}{2} & \text{si } x\le 1\\ f_2(x)=\ln|x| & \text{si } x>1\end{cases}$

b) La pendiente de la recta tangente en el punto *x* = 2 es $f'(2)=\frac{1}{2}$. Además, $f(2)=\ln 2$. Por tanto, la ecuación de la recta es: $y=\frac{1}{2}x+n\rightarrow \ln 2=\frac{1}{2}\cdot 2+n\rightarrow n=\ln 2-1\rightarrow y=\frac{1}{2}x+\ln 2-1$

69. Página 288

a) $\int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + k = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + k$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

b) $\int \ln(2x+1) dx = x \ln|2x+1| - \int \frac{2x}{2x-1} dx = x \ln|2x+1| - \int \left(1 - \frac{1}{2x+1}\right) dx =$

$u = \ln(2x+1) \rightarrow du = \frac{2}{2x+1} dx$

$= x \ln|2x+1| - x + \frac{\ln|2x+1|}{2} + k = \left(x + \frac{1}{2}\right) \ln|2x+1| - x + k$

c) $I = \int e^{-x} sen 2x dx = -e^{-x} \frac{\cos 2x}{2} - \frac{1}{2} \int e^{-x} \cos 2x \, dx = -e^{-x} \frac{\cos 2x}{2} - \frac{1}{2}\left(e^{-x} \frac{sen 2x}{2} + \frac{1}{2} \int e^{-x} sen 2x \, dx\right) =$

$u = e^{-x} \rightarrow du = -e^{-x} dx$

$dv = sen 2x \, dx \rightarrow v = \frac{-\cos 2x}{2}$

$u = e^{-x} \rightarrow du = -e^{-x} dx$

$dv = \cos 2x \, dx \rightarrow v = \frac{sen 2x}{2}$

$= -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{sen 2x}{4} - \frac{1}{4} \int e^{-x} sen 2x \, dx = -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{sen 2x}{4} - \frac{1}{4} I$

$I = -e^{-x} \frac{\cos 2x}{2} - e^{-x} \frac{sen 2x}{4} - \frac{1}{4} I \rightarrow \frac{5}{4} I = -e^{-x}\left(\frac{\cos 2x}{2} + \frac{sen 2x}{4}\right) \rightarrow I = \frac{-e^{-x}}{5}(2\cos 2x + sen 2x) + k$

d) $\int arc\,tg\, x \, dx = x \cdot arc\,tg\, x - \int \frac{x}{x^2+1} dx = x \cdot arc\,tg\, x - \frac{\ln|x^2+1|}{2} + k$

$u = arc\,tg\, x \rightarrow du = \frac{1}{x^2+1} dx$

e) $I = \int \frac{\ln x}{x} dx = \ln^2 x - \int \frac{\ln x}{x} dx = \ln^2 x - I \rightarrow I = \ln^2 x - I \rightarrow I = \int \frac{\ln x}{x} dx = \frac{1}{2} \ln^2 x + k$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

f) $\int x \, sen 2x \, dx = \frac{-x \cos 2x}{2} + \int \frac{\cos 2x}{2} dx = -\frac{x \cos 2x}{2} + \frac{sen 2x}{4} + k$

$u = x \rightarrow du = dx$

g) $\int x^2 sen 2x \, dx = \frac{-x^2 \cos 2x}{2} + \int x \cos 2x \, dx = \frac{-x^2 \cos 2x}{2} + \frac{x \, sen 2x}{2} - \int \frac{sen 2x}{2} dx =$

$u = x^2 \rightarrow du = 2x \, dx$

$dv = sen 2x \, dx \rightarrow v = \frac{-\cos 2x}{2}$

$u = x \rightarrow du = dx$

$dv = \cos 2x \, dx \rightarrow v = \frac{sen 2x}{2}$

$= \frac{-x^2 \cos 2x}{2} + \frac{x \, sen 2x}{2} + \frac{\cos 2x}{4} + k$

h) $\int (2x+3) e^{2x} dx = \frac{(2x+3)e^{2x}}{2} - \int e^{2x} dx = \frac{(2x+3)e^{2x}}{2} - \frac{e^{2x}}{2} + k = (x+1)e^{2x} + k$

$u = 2x + 3 \rightarrow du = 2x \, dx$

i) $\int \frac{x}{e^x}dx = -\frac{x}{e^x} + \int e^{-x}\,dx = -\frac{x}{e^x} - \frac{1}{e^x} + k = \frac{-x-1}{e^x} + k$

$u = x \rightarrow du = dx$

j) $\int (x^2-5)\cos x\,dx = (x^2-5)\text{sen}\,x - 2\int x\,\text{sen}\,x\,dx = (x^2-5)\text{sen}\,x + 2x\cos x - 2\int \cos x\,dx =$

$u = x^2-5 \rightarrow du = 2x\,dx$

$u = x \rightarrow du = dx$
$dv = \text{sen}\,x\,dx \rightarrow v = -\cos$

$= (x^2-5)\text{sen}\,x + 2x\cos x - 2\,\text{sen}\,x + k = (x^2-7)\,\text{sen}\,x + 2x\cos x + k$

k) $\int (2x^2+x-2)e^{3x}\,dx = \frac{(2x^2+x-2)e^{3x}}{3} - \int \frac{(4x+1)e^{3x}}{3}dx = \frac{(2x^2+x-2)e^{3x}}{3} - \frac{1}{3}\left(\frac{(4x+1)e^{3x}}{3} - \int \frac{4e^{3x}}{3}dx\right) =$

$u = 2x^2+x-2 \rightarrow du = (4x+1$

$u = 4x+1 \rightarrow du = 4\,dx$

$= \frac{(2x^2+x-2)e^{3x}}{3} - \frac{(4x+1)e^{3x}}{9} + \frac{4}{27}e^{3x} + k = \frac{1}{27}e^{3x}(18x^2-3x-17) + k$

l) $\int (2+e^{2x})\cos(x+1)\,dx = \int 2\cos(x+1)\,dx + \int e^{2x}\cos(x+1)\,dx = 2\,\text{sen}(x+1) + e^{2x}\text{sen}(x+1) - \int 2e^{2x}\text{sen}(x+1)\,dx =$

$u = e^{2x} \rightarrow du = 2e^{2x}\,dx$
$dv = \cos(x+1)\,dx \rightarrow v = \text{sen}(x$

$u = e^{2x} \rightarrow du = 2e^{2x}\,dx$
$dv = \text{sen}(x+1)\,dx \rightarrow v = -\cos$

$= 2\,\text{sen}(x+1) + e^{2x}\,\text{sen}(x+1) + 2e^{2x}\cos(x+1) - 2\int 2e^{2x}\cos(x+1)\,dx = 2\,\text{sen}(x+1) + \frac{e^{2x}\,\text{sen}(x+1) + 2e^{2x}\cos(x+1)}{5} + k$

70. Página 288

a) $\int \ln x^2\,dx = x\ln x^2 - \int x\cdot\frac{2x}{x^2}\,dx = x\ln x^2 - \int 2\,dx = x\ln x^2 - 2x + k$

$u = \ln x^2 \rightarrow du = \frac{2x}{x^2}dx$

b) $\int \ln x^3\,dx = x\ln x^3 - \int x\cdot\frac{3x^2}{x^3}\,dx = x\ln x^3 - \int 3\,dx = x\ln x^3 - 3x + k$

$u = \ln x^3 \rightarrow du = \frac{3x^2}{x^3}dx$

c) $\int \ln x^4\,dx = x\ln x^4 - \int x\cdot\frac{4x^3}{x^4}\,dx = x\ln x^4 - \int 4\,dx = x\ln x^4 - 4x + k$

$u = \ln x^4 \rightarrow du = \frac{4x^3}{x^4}dx$

d) $\int \ln x^5\,dx = x\ln x^5 - \int x\cdot\frac{5x^4}{x^5}\,dx = x\ln x^5 - \int 5\,dx = x\ln x^5 - 5x + k$

$u = \ln x^5 \rightarrow du = \frac{5x^4}{x^5}dx$

e) $\int \ln x^n\,dx = x\ln x^n - \int x\cdot\frac{nx^{n-1}}{x^n}\,dx = x\ln x^n - \int n\,dx = x\ln x^n - nx + k$

$u = \ln x^n \rightarrow du = \frac{nx^{n-1}}{x^n}dx$

71. Página 288

a) $\int x^2 \cdot e^{3x}\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx\right] = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\cdot\frac{1}{3}e^{3x} + k =$

$u = x^2 \rightarrow du = 2x\,dx$

$u = x \rightarrow du = dx$

$dv = e^{3x}dx \rightarrow v = \frac{1}{3}$

$= \frac{e^{3x}}{3}\left(x^2 - \frac{2}{3}x + \frac{2}{9}\right) + k$

b) $\int \ln\left(\frac{x+1}{x-1}\right)^x dx = \int x\ln\left(\frac{x+1}{x-1}\right)dx = \int x\ln(x+1)dx - \int x\ln(x-1)dx = \frac{x}{x+1} - \int\frac{1}{x+1}dx - \frac{x}{x-1} + \int\frac{1}{x-1}dx =$

$u = x \rightarrow du = dx$

$dv = \ln(x+1)dx \rightarrow v =$

$u = x \rightarrow du = dx$

$dv = \ln(x-1)dx \rightarrow v =$

$= \frac{x}{x+1} - \ln|x+1| - \frac{x}{x-1} + \ln|x-1| + k$

c) $\int \frac{\ln x}{x^3}dx = -\frac{\ln x}{2x^2} + \frac{1}{2}\int\frac{1}{x^3}dx = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + k$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

d) $\int x \cdot \text{arc}\,\text{tg}(x+1)dx = \frac{1}{2}x^2\text{arc}\,\text{tg}(x+1) - \int\frac{x^2}{2(x^2+2x+2)}dx = \frac{1}{2}x^2\text{arc}\,\text{tg}(x+1) - \frac{1}{2}\int\left(1 - \frac{2(x+1)}{x^2+2x+2}\right)dx =$

$u = \text{arc tg}(x+1) \rightarrow du = \frac{1}{1+(x+1)^2}dx$

$= \frac{1}{2}x^2\text{arc}\,\text{tg}(x+1) - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{2(x+1)}{x^2+2x+2}dx = \frac{1}{2}x^2\text{arc}\,\text{tg}(x+1) - \frac{1}{2}x + \frac{1}{2}\ln|x^2+2x+2| + k$

e) $\int x\cdot 2^x dx = x\cdot\frac{2^x}{\ln 2} - \int\frac{2^x}{\ln 2}dx = x\cdot\frac{2^x}{\ln 2} - \frac{1}{\ln 2}\int 2^x dx = x\cdot\frac{2^x}{\ln 2} - \frac{1}{\ln 2}\cdot\frac{1}{\ln 2}\int \ln 2\cdot 2^x dx = \frac{2^x x}{\ln 2} - \frac{2^x}{\ln^2 2} + k$

$u = x \rightarrow du = dx$

f) $\int x\cdot\ln^2 x\,dx = \ln^2 x\frac{x^2}{2} - \int x\ln x\,dx = \ln^2 x\cdot\frac{x^2}{2} - \left(\frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx\right) = \ln^2 x\cdot\frac{x^2}{2} - \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + k =$

$u = \ln^2 x \rightarrow du = \frac{2\ln x}{x}$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$= \frac{1}{2}x^2\left(\ln^2 x - \ln x - \frac{1}{2}\right) + k$

g) $\int \text{arc sen}\,x\,dx = x\,\text{arc sen}\,x - \int\frac{x}{\sqrt{1-x^2}}dx = x\,\text{arc sen}\,x + \frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}dx = x\,\text{arc sen}\,x + \sqrt{1-x^2} + k$

$u = \text{arc sen}\,x \rightarrow du = \frac{1}{\sqrt{1-x^2}}dx$

h) $\int\frac{x\cdot\text{arc sen}\,x}{\sqrt{1-x^2}}dx = \int t\,\text{sen}\,t\,dt = -t\cos t + \int\cos t\,dt = -t\cos t + \text{sen}\,t = -\cos(\text{arc sen}\,x)\cdot\text{arc sen}\,x + x + k =$

$t = \text{arc sen}\,x \rightarrow dt = \frac{1}{\sqrt{1-x^2}}$

$u = t \rightarrow du = dt$

$= -\text{arc sen}\,x\sqrt{1-x^2} + x + k$

72. Página 288

a) $\int \sqrt[3]{x}\ln x\,dx = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{3}{4}\int \sqrt[3]{x}\,dx = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{3}{4}\cdot\frac{3}{4}\sqrt[3]{x^4} = \frac{3}{4}\ln x\sqrt[3]{x^4} - \frac{9}{16}\sqrt[3]{x^4}$

$u = \ln x \to du = \frac{1}{x}dx$

b) $\int (x^2+3x)e^{-x+7}\,dx = -(x^2+3x)e^{-x+7} + \int (2x+3)e^{-x+7}\,dx = -(x^2+3x)e^{-x+7} - (2x+3)e^{-x+7} - 2e^{-x+7} + k$

$u = x^2+3x \to du = (2x+3$

$u = 2x+3 \to du = 2dx$
$dv = e^{-x+7}\,dx \to v = -e^{-x+7}$

c) $\int e^x\cos(3x)dx = \frac{1}{3}e^x sen3x - \frac{1}{3}\int e^x sen3x\,dx = \frac{1}{3}e^x sen3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x + \frac{1}{3}\int e^x\cos 3x\,dx\right) =$

$u = e^x \to du = e^x dx$
$dv = \cos 3x\,dx \to v = \frac{1}{3}sen$

$u = e^x \to du = e^x dx$
$dv = sen3x\,dx \to v = -\frac{1}{3}c$

$= \frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x - \frac{1}{9}\int e^x\cos 3x\,dx$

$I = \frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x - \frac{1}{9}I \to I = \frac{9}{10}\left(\frac{1}{3}e^x sen3x + \frac{1}{9}\cos 3x\right) \to I = \frac{3}{10}e^x sen3x + \frac{1}{10}\cos 3x$

d) $\int \ln(1+x^2)dx = x\ln(1+x^2) - \int\frac{2x^2}{x^2+1}dx = x\ln(1+x^2) - \left[\int\left(2-\frac{2}{x^2+1}\right)dx\right] = x\ln(1+x^2) - 2x - 2arc\,tg\,x + k$

$u = \ln(1+x^2) \to du = \frac{2x}{1+x^2}$

e) $\int x^2\cdot arc\,tg\,x\,dx = \frac{x^3}{3}arc\,tg\,x - \int\frac{x^3}{3(x^2+1)}dx = \frac{x^3}{3}arc\,tg\,x - \frac{1}{3}\int\left(x - \frac{x}{x^2+1}\right)dx = \frac{x^3}{3}arc\,tg\,x - \frac{1}{6}x^2 + \frac{1}{6}\ln|x^2+1| + k$

$u = arc\,tg\,x \to du = \frac{1}{x^2+1}d$

f) $\int\frac{\sqrt{x}+1}{\sqrt[3]{x}}\ln x\,dx = \int\left(x^{\frac{1}{6}} + x^{-\frac{1}{3}}\right)\ln x\,dx = \ln x\left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right) - \int\frac{\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}}{x}dx =$

$u = \ln x \to du = \frac{1}{x}dx$
$dv = (x^{1/6} + x^{-1/3})dx \to v = \frac{6}{7}x^{7/6}$

$= \ln x\left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right) - \int\left(\frac{6}{7}x^{1/6} + \frac{3}{2}x^{-1/3}\right)dx = \left(\frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3}\right)\ln x - \frac{36}{49}x^{7/6} - \frac{9}{4}x^{2/3} + k$

73. Página 288

$\int x\cdot sen(2x+1)\cdot\cos(2x-1)dx = \frac{1}{2}\int x(sen(2) + sen(4x))dx = \frac{1}{2}sen(2)\int x\,dx + \frac{1}{2}\int x\,sen(4x)dx =$

$= \frac{1}{4}sen(2)x^2 + \frac{1}{2}\left(-\frac{1}{4}x\cos(4x) + \frac{1}{4}\int\cos(4x)dx\right) = \frac{1}{4}x^2 sen(2) - \frac{1}{8}x\cos(4x) + \frac{1}{32}sen(4x) + k$

$u = x \to du = dx$
$dv = sen4x\,dx \to v = -\frac{1}{4}\cos 4x$

74. Página 288

$$f(x)=\int x^2\cdot \operatorname{sen} x\,dx = -x^2\cos x+2\int x\cos x\,dx = -x^2\cos x+2\left(x\operatorname{sen} x-\int \operatorname{sen} x\,dx\right)=$$

$u=x^2 \rightarrow du=2x\,dx$
$dv=\operatorname{sen} x\,dx \rightarrow v=-c$

$u=x \rightarrow du=dx$
$dv=\cos x\,dx \rightarrow v=se$

$$=-x^2\cos x+2x\operatorname{sen} x+2\cos x+k$$

$$f(0)=1 \rightarrow 2\cos(0)+k=1 \rightarrow k=1-2=-1$$

$$f(x)=-x^2\cos x+2x\operatorname{sen} x+2\cos x-1$$

75. Página 288

$$f'(x)=\int f''(x)dx=\int 2x\cdot\ln x dx=2\left(\frac{1}{2}x^2\ln x-\int\frac{1}{2}x\,dx\right)=x^2\ln x-\frac{1}{2}x^2+k$$

$u=\ln x \rightarrow du=\frac{1}{x}dx$

$$f'(1)=0 \rightarrow \ln(1)-\frac{1}{2}+k=0 \rightarrow k=k=\frac{1}{2} \qquad f'(x)=x^2\ln x-\frac{1}{2}x^2+\frac{1}{2}$$

$$f(x)=\int f'(x)=\int\left(x^2\ln x-\frac{1}{2}x^2+\frac{1}{2}\right)dx=\int x^2\ln x dx-\frac{1}{2}\int x^2dx+\frac{1}{2}\int 1dx=\frac{1}{3}x^3\ln x-\frac{1}{3}\int x^2dx-\frac{1}{6}x^3+\frac{1}{2}x=$$

$$=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3-\frac{1}{6}x^3+\frac{1}{2}x+k=\left(\frac{1}{3}\ln x-\frac{5}{18}\right)x^3+\frac{1}{2}x+k$$

$u=\ln x \rightarrow du=\frac{1}{x}dx$

$$f(e)=\frac{e}{2} \rightarrow \left(\frac{1}{3}-\frac{5}{18}\right)e^3+\frac{1}{2}e+k=\frac{e}{2} \rightarrow k=-\frac{1}{18}e^3$$

$$f(x)=\left(\frac{1}{3}\ln x-\frac{5}{18}\right)x^3+\frac{1}{2}x-\frac{1}{18}e^3$$

76. Página 288

$$F(x)=\int f(x)dx=\int(ax^2+x\cdot\cos x+1)dx=a\int x^2\,dx+\int x\cos x\,dx+\int dx=\frac{a}{3}x^3+x\operatorname{sen} x-\int \operatorname{sen} x\,dx+x=$$

$u=x \rightarrow du=dx$
$dv=\cos x\,dx \rightarrow v=\operatorname{sen} x$

$$=\frac{a}{3}x^3+x\operatorname{sen}x+\cos x+x+k$$

Tomamos $k=0$: $F(\pi)=\pi \rightarrow \frac{a}{3}\pi^3-1+\pi=\pi \rightarrow a=\frac{3}{\pi^3}$

77. Página 288

a) $f(x)=\int f'(x)dx=\int x^2\operatorname{sen} x\,dx=-x^2\cos x+2\int x\cos x\,dx=-x^2\cos x+2x\operatorname{sen} x+2\cos x+k=$

$u=x^2 \rightarrow du=2xdx$
$dv=\operatorname{sen} x\,dx \rightarrow v=-c$

$u=x \rightarrow du=dx$
$dv=\cos x\,dx \rightarrow v=se$

$$=(2-x^2)\cos x+2x\operatorname{sen} x+k$$

$$f(0)=1 \rightarrow 2+k=1 \rightarrow k=-3 \rightarrow f(x)=(2-x^2)\cos x+2x\operatorname{sen} x-3$$

b) $f(x)=\int f'(x)dx=\int x\ln x\,dx=\frac{x^2\ln x}{2}-\int\frac{x}{2}dx=\frac{x^2\ln x}{2}-\frac{x^2}{4}+k$

$u=\ln x\rightarrow du=\frac{1}{x}dx$

$f(1)=\frac{1}{2}\rightarrow -\frac{1}{4}+k=\frac{1}{2}\rightarrow k=\frac{3}{4}\rightarrow f(x)=\frac{x^2\ln x}{2}-\frac{x^2}{4}+\frac{3}{4}$

78. Página 288

La pendiente de la recta tangente es el valor de la derivada de la función.

$$F(x)=\int(2x+3)e^{2x}\,dx=\frac{(2x+3)e^{2x}}{2}-\int e^{2x}\,dx=\frac{(2x+3)e^{2x}}{2}-\frac{e^{2x}}{2}+k=$$

$$=\frac{(2x+3)e^{2x}-e^{2x}}{2}+k=(x+1)e^{2x}+k\;\rightarrow\;F(0)=1\rightarrow 1+k=1\rightarrow k=0\rightarrow F(x)=(x+1)e^{2x}$$

79. Página 288

$$F(x)=\int f(x)dx=\int\frac{3}{1+x^2}dx=3\,\mathrm{arc\,tg}(x)+k$$

$$F(1)=\frac{\pi}{2}\rightarrow 3\,\mathrm{arc\,tg}(1)+k=\frac{\pi}{2}\rightarrow k=\frac{\pi}{2}-\frac{3\pi}{4}\rightarrow k=-\frac{\pi}{4}\rightarrow F(x)=3\,\mathrm{arc\,tg}(x)-\frac{\pi}{4}$$

80. Página 288

a) $\int\frac{3}{x^2-3x+2}dx=\int\left(\frac{-3}{x-1}+\frac{3}{x-2}\right)dx=\int\frac{-3}{x-1}dx+\int\frac{3}{x-2}dx=-3\ln|x-1|+3\ln|x-2|+k$

b) $\int\frac{2-x}{x^2+3x}dx=\int\left(\frac{\frac{2}{3}}{x}+\frac{\frac{-5}{3}}{x+3}\right)dx=\frac{2}{3}\int\frac{1}{x}dx+\frac{-5}{3}\int\frac{1}{x+3}dx=\frac{2}{3}\ln|x|-\frac{5}{3}\ln|x+3|+k$

c) $\int\frac{x-3}{x^2-4}dx=\int\left(\frac{\frac{5}{4}}{x+2}+\frac{\frac{-1}{4}}{x-2}\right)dx=\frac{5}{4}\int\frac{1}{x+2}dx-\frac{1}{4}\int\frac{1}{x-2}dx=\frac{5}{4}\ln|x+2|-\frac{1}{4}\ln|x-2|+k$

d) $\int\frac{x}{x^2+6x+5}dx=\int\left(\frac{-\frac{1}{4}}{x+1}+\frac{\frac{5}{4}}{x+5}\right)dx=-\frac{1}{4}\int\frac{1}{x+1}dx+\frac{5}{4}\int\frac{1}{x+5}dx=-\frac{1}{4}\ln|x+1|+\frac{5}{4}\ln|x+5|+k$

81. Página 288

a) $\int f(x)dx=\int\frac{3}{x^2-2x+1}dx=3\int\frac{1}{(x-1)^2}dx=\frac{-3}{x-1}+k$

b) $\int g(x)dx=\int\frac{x+2}{x^2-2x+1}dx=\int\left(\frac{1}{(x-1)}+\frac{3}{(x-1)^2}\right)dx=\int\frac{1}{(x-1)}dx+\int\frac{3}{(x-1)^2}dx=\ln|x-1|-\frac{3}{x-1}+k$

c) $\int h(x)dx=\int\frac{x}{x^2-2x+1}dx=\int\left(\frac{1}{(x-1)}+\frac{1}{(x-1)^2}\right)dx=\int\frac{1}{(x-1)}dx+\int\frac{1}{(x-1)^2}dx=\ln|x-1|-\frac{1}{x-1}+k$

d) $\int i(x)dx=\int\frac{x^2}{x^2-2x+1}dx=\int\left(1+\frac{2x-1}{x^2-2x+1}\right)dx=\int 1dx+\int\frac{2x-1}{x^2-2x+1}dx=x+\int\left(\frac{2}{x-1}+\frac{1}{(x-1)^2}\right)dx=$

$=x+2\ln|x-1|-\frac{1}{x-1}+k$

82. Página 289

a) $\int \frac{2}{x^3+x^2}dx = \int\left(-\frac{2}{x}+\frac{2}{x^2}+\frac{2}{x+1}\right)dx = -2\ln|x| - \frac{2}{x} + 2\ln|x+1| + k$

b) $\int \frac{1+x^2}{3x^2-x^3}dx = \int\left(\frac{\frac{1}{9}}{x}+\frac{\frac{1}{3}}{x^2}+\frac{\frac{10}{9}}{3-x}\right)dx = \frac{1}{9}\ln|x| - \frac{1}{3x} - \frac{10}{9}\ln|3-x| + k$

c) $\int \frac{x+2}{x^3-x^2-x+1}dx = \int\left(\frac{-\frac{1}{4}}{x-1}+\frac{\frac{3}{2}}{(x-1)^2}+\frac{\frac{1}{4}}{x+1}\right)dx = -\frac{1}{4}\ln|x-1| - \frac{3}{2}\cdot\frac{1}{x-1} + \frac{1}{4}\ln|x+1| + k$

d) $\int \frac{1+x^3}{x^3-2x^2}dx = \int\left(1+\frac{2x^2+1}{x^3-2x^2}\right)dx = \int 1dx + \int\left(\frac{\frac{-1}{4}}{x}+\frac{\frac{-1}{2}}{x^2}+\frac{\frac{9}{4}}{x-2}\right)dx = x - \frac{1}{4}|\ln x| + \frac{1}{2x} + \frac{9}{4}\ln|x-2| + k$

83. Página 289

a) $\int \frac{2}{x^4-x^2}dx = \int\left(\frac{-2}{x^2}+\frac{-1}{x+1}+\frac{1}{x-1}\right)dx = \frac{2}{x} - \ln|x+1| + \ln|x-1| + k$

b) $\int \frac{x+1}{4x^2-x^4}dx = \int\left(\frac{1}{4x}+\frac{1}{4x^2}+\frac{3}{16(2-x)}-\frac{1}{16(2+x)}\right)dx = \frac{1}{4}\ln|x| - \frac{1}{4x} - \frac{3}{16}\ln|2-x| - \frac{1}{16}\ln|2+x| + k$

c) $\int \frac{x+6}{x^4-3x^3+x^2+3x-2}dx = \int\left(-\frac{9}{4(x-1)}-\frac{5}{12(x+1)}-\frac{7}{2(x-1)^2}+\frac{8}{3(x-2)}\right)dx =$

$= -\frac{9}{4}\ln|x-1| - \frac{5}{12}\ln|x+1| + \frac{7}{2(x-1)} + \frac{8}{3}\ln|x-2| + k$

84. Página 289

a) $\int \frac{2}{x^3+x}dx = \int\left(\frac{2}{x}-\frac{2x}{x^2+1}\right)dx = 2\ln|x| - \ln|x^2+1| + k$

b) $\int \frac{x+2}{x^3+x^2+x+1}dx = \int\left(\frac{3}{2(x^2+1)}-\frac{x}{2(x^2+1)}+\frac{1}{2(x+1)}\right)dx = \frac{3}{2}arctg x - \frac{1}{4}\ln|x^2+1| + \frac{1}{2}\ln|x+1| + k$

c) $\int \frac{x+3}{4x^3-4x^2+x-1}dx = \int\left(\frac{-16x-11}{5(4x^2+1)}+\frac{4}{5(x-1)}\right)dx = \int\left(\frac{-16x}{5(4x^2+1)}-\frac{11}{5(4x^2+1)}+\frac{4}{5(x-1)}\right)dx =$

$= -\frac{2}{5}\ln|4x^2+1| - \frac{11}{10}arctg(2x) + \frac{4}{5}\ln|x-1| + k$

d) $\int \frac{1}{(x-2)^2(x^2+2)}dx = \int\left(\frac{2x}{18(x^2+2)}+\frac{1}{18(x^2+2)}-\frac{1}{9(x-2)}+\frac{1}{6(x-2)^2}\right)dx =$

$= \frac{1}{18}\ln|x^2+2| - \frac{1}{6(x-2)} - \frac{1}{9}\ln|x-2| + \frac{\sqrt{2}}{36}arctg\left(\frac{x}{\sqrt{2}}\right) + k$

e) $\int \frac{3x^2+1}{x^4-1}dx = \int\left(\frac{1}{x^2+1}-\frac{1}{x+1}+\frac{1}{x-1}\right)dx = \ln|1-x| - \ln|x+1| + arctg\, x + k$

f) $\int \frac{x^2+x+1}{x^3-x^2-x+1}dx = \int\left(\frac{3}{4(x-1)}+\frac{3}{2(x-1)^2}+\frac{1}{4(x+1)}\right)dx = \frac{3}{4}\ln|x-1| - \frac{3}{2(x-1)} + \frac{1}{4}\ln|x+1| + k$

g) $\int \frac{x^2+1}{(x-1)^3}dx = \int\left(\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx = \ln|x-1| - \frac{2}{(x-1)} - \frac{1}{(x-1)^2} + k = \ln|x-1| + \frac{1-2x}{(x-1)^2} + k$

85. Página 289

a) $\int \frac{x+3}{x-1}dx = \int\left(1+\frac{4}{x-1}\right)dx = x+4\ln|x-1|+k$

b) $\int \frac{2x+3}{2x+1}dx = \int\left(1+\frac{2}{2x+1}\right)dx = x+\ln|2x+1|+k$

c) $\int \frac{3}{x^2+x-2}dx = \int\left(\frac{1}{x-1}-\frac{1}{x+2}\right)dx = \ln|x-1|-\ln|x+2|+k$

d) $\int \frac{5x-1}{x^2-1}dx = \int\left(\frac{2}{x-1}+\frac{3}{x+1}\right)dx = 2\ln|x-1|+3\ln|x+1|+k$

e) $\int \frac{x-2}{x^2-x}dx = \int\left(\frac{-1}{x-1}+\frac{2}{x}\right)dx = -\ln|x-1|+2\ln|x|+k$

f) $\int \frac{x+2}{x^2-1}dx = \int\left(\frac{3}{2(x-1)}-\frac{1}{2(x+1)}\right)dx = \frac{3}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+k$

g) $\int \frac{1}{x^2+9}dx = \frac{1}{3}arc\,tg\,\frac{x}{3}+k$

h) $\int \frac{12}{x^3+4x^2+x-6}dx = \int\left(\frac{1}{x-1}-\frac{4}{x+2}+\frac{3}{x+3}\right)dx = \ln|x-1|-4\ln|x+2|+3\ln|x+3|+k$

86. Página 289

a) $\int \frac{1}{x^2-5x+6}dx = \int\left(\frac{1}{x-3}-\frac{1}{x-2}\right)dx = \ln|x-3|-\ln|x-2|+k$

b) $\int \frac{x-2}{x^2+2x-3}dx = \int \frac{1}{4}\left(\frac{5}{x+3}-\frac{1}{x-1}\right)dx = \frac{5}{4}\ln|x+3|-\frac{1}{4}\ln|x-1|+k$

c) $\int \frac{x-4}{x^2+2x-3}dx = \int \frac{1}{4}\left(\frac{7}{x+3}-\frac{3}{x-1}\right)dx = \frac{7}{4}\ln|x+3|-\frac{3}{4}\ln|x-1|+k$

d) $\int \frac{2x+8}{x^2-4}dx = \int\left(\frac{3}{x-2}-\frac{1}{x+2}\right)dx = 3\ln|x-2|-\ln|x+2|+k$

e) $\int \frac{x^2}{x-4}dx = \int\left(x+4+\frac{16}{x-4}\right)dx = \frac{x^2}{2}+4x+16\ln|x-4|+k$

f) $\int \frac{x}{x^4+1}dx = \frac{1}{2}arc\,tg\,x^2+k$

g) $\int \frac{x^4-x^3-x+1}{x^3-x^2}dx = \int\left(x-\frac{1}{x^2}\right)dx = \frac{x^2}{2}+\frac{1}{x}+k$

h) $\int \frac{x^3}{(x+1)^4}dx = \int\left(\frac{-1}{(x+1)^4}+\frac{3}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{1}{x+1}\right)dx =$

$= \frac{1}{3(x+1)^3}-\frac{3}{2(x+1)^2}+\frac{3}{x+1}+\ln|x+1|+k$

i) $\int \frac{2x+5}{(x+3)^3}dx = \int\left(\frac{-1}{(x+3)^3}+\frac{2}{(x+3)^2}\right)dx = \frac{1}{2(x+3)^2}-\frac{2}{x+3}+k$

j) $\int \frac{2x}{(x+1)^2}dx = \int\left(\frac{-2}{(x+1)^2}+\frac{2}{x+1}\right)dx = \frac{2}{x+1}+2\ln|x+1|+k$

87. Página 289

a) $\int \frac{4x^3+2x-1}{2x+1}dx = \int\left(2x^2 - x - \frac{5}{2(2x+1)} + \frac{3}{2}\right)dx = \frac{2}{3}x^3 - \frac{1}{2}x^2 - \frac{5}{4}\ln|2x+1| + \frac{3}{2}x + k$

b) $\int \frac{-x^2+x-1}{3-x}dx = \int\left(x+2-\frac{7}{3-x}\right)dx = \frac{1}{2}x^2 + 2x + 7\ln|3-x| + k$

c) $\int \frac{x^3}{x^2+4x-5}dx = \int\left(x+\frac{1}{6(x-1)}+\frac{125}{6(x+5)}-4\right)dx = \frac{1}{2}(x-8)x + \frac{1}{6}\ln|1-x| + \frac{125}{6}\ln|x+5| + k$

d) $\int \frac{x^3-x+6}{x^2+5x+4}dx = \int\left(x+\frac{2}{x+1}+\frac{18}{x+4}-5\right)dx = \frac{1}{2}x^2 + 2\ln|x+1| + 18\ln|x+4| - 5x + k$

88. Página 289

a) $\int x^2\cdot\sqrt{x^3+3}\,dx = \frac{1}{3}\int\sqrt{t}dt = \frac{1}{3}\cdot\frac{2}{3}\sqrt{t^3} + k = \frac{2}{9}\sqrt{t^3} + k = \frac{2}{9}\sqrt{(x^3+3)^3} + k$

$t = x^3+3 \rightarrow dt = 3x^2dx \rightarrow \frac{1}{3x^2}dt = dx$

b) $\int x^3\cdot e^{x^4+1}dx = \int\frac{1}{4}e^t dt = \frac{1}{4}e^t + k = \frac{1}{4}e^{x^4+1} + k$

$t = x^4+1 \rightarrow dt = 4x^3dx \rightarrow \frac{1}{4}dt = x^3dx$

c) $\int \frac{2}{x\cdot\ln x}dx = 2\int\frac{1}{t}dt = 2\ln|t| + k = 2\ln|\ln x| + k$

$t = \ln x \rightarrow dt = \frac{1}{x}dx$

d) $\int \frac{\ln x^2}{x}dx = \int\frac{2\ln x}{x}dx = 2\int\frac{\ln x}{x}dx = 2\int t\,dt = 2\frac{t^2}{2} + k = \ln^2 x + k$

$t = \ln x \rightarrow dt = \frac{1}{x}dx$

89. Página 289

Hacemos en todos los apartados el cambio de variable:

$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}dx \rightarrow 2dt = \frac{1}{\sqrt{x}}dx$

a) $\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}}dx = 2\int e^{t+1}dt = 2e^{t+1} + k = 2e^{\sqrt{x}+1} + k$

b) $\int \frac{sen\sqrt{x}}{\sqrt{x}}dx = 2\int sen t\,dt = -2cos\,t + k = -2cos(\sqrt{x}) + k$

c) $\int \frac{e^{-\sqrt{x}+1}}{\sqrt{x}}dx = 2\int e^{-t+1}dt = -2e^{-t+1} + k = -2e^{-\sqrt{x}+1} + k$

d) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = 2\int\sqrt{1+t}\,dt = 2\int(1+t)^{\frac{1}{2}}dt = \frac{4\sqrt{(1+t)^3}}{3} + k = \frac{4\sqrt{(1+\sqrt{x})^3}}{3} + k$

90. Página 289

a) $\int \cos x \, sen^3 x \, dx = \int t^3 \, dt = \frac{t^4}{4} + k = \frac{sen^4 x}{4} + k$

$t = sen\, x \rightarrow dt = \cos x \, dx$

b) $\int x \ln(1+x^2) dx = \frac{1}{2}\int \ln t \, dt = \frac{1}{2}(t \ln t - t) + k = \frac{1}{2}\left((1+x^2)\ln(1+x^2) - (1+x^2)\right) + k$

$t = 1 + x^2 \rightarrow dt = 2x dx$

c) $\int \frac{\ln 2x}{x} dx = \int t \, dt = \frac{t^2}{2} + k = \frac{\ln^2(2x)}{2} + k$

$t = \ln 2x \rightarrow dt = \frac{1}{x} dx$

d) $\int 2x \, sen x^2 \, dx = \int sen\, t \, dt = -\cos t + k = -\cos x^2 + k$

$t = x^2 \rightarrow dt = 2x \, dx$

e) $\int x\sqrt{x+1} dx = \int 2(t^2-1)t^2 \, dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + k = \sqrt{x+1}\left(\frac{2(x+1)^2}{5} - \frac{2(x+1)}{3}\right) + k$

$t = \sqrt{x+1} \rightarrow t^2 = x+1 \rightarrow 2t \, dt = dx$

f) $\int \frac{2x}{x^2-1} dx = \int \frac{1}{t} dt = \ln|t| + k = \ln|x^2-1| + k$

$t = x^2 - 1 \rightarrow dt = 2x \, dx$

g) $\int \cos^2 x \, sen\, x \, dx = \int -t^2 \, dt = \frac{-t^3}{3} + k = \frac{-\cos^3 x}{3} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

h) $\int sen\, x \, e^{\cos x} dx = \int -e^t \, dt = -e^t + k = -e^{\cos x} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

i) $\int \cos^2 x \, sen^3 x \, dx = \int -t^2(1-t^2) dt = -\frac{t^3}{3} + \frac{t^5}{5} + k = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

j) $\int \frac{sen^3 x}{\cos x} dx = \int \frac{t^2-1}{t} dt = \frac{t^2}{2} - \ln|t| + k = \frac{\cos^2 x}{2} - \ln|\cos x| + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

k) $\int (x^2+1)e^{x^3+3x} dx = \int \frac{e^t}{3} dt = \frac{e^t}{3} + k = \frac{e^{x^3+3x}}{3} + k$

$t = x^3 + 3x \rightarrow dt = (3x^2+3) dx$

l) $\int \cos^5 x \, sen^3 x \, dx = \int t^5(t^2-1) dt = \frac{t^8}{8} - \frac{t^6}{6} + k = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + k$

$t = \cos x \rightarrow dt = -sen\, x \, dx$

91. Página 289

a) $\int \frac{2}{4+x^2}dx = \int \frac{1}{1+t^2}dt = arc\,tg\,t + k = arc\,tg\frac{x}{2} + k$

$$t = \frac{x}{2} \rightarrow dt = \frac{1}{2}dx$$

b) $\int x(x+5)^{10}dx = \int (t-5)t^{10}dt = \int (t^{11} - 5t^{10})dt = \frac{t^{12}}{12} - \frac{5t^{11}}{11} + k = \frac{(x+5)^{12}}{12} - \frac{5(x+5)^{11}}{11} + k$

$$t = x+5 \rightarrow dt = dx$$

c) $\int \frac{tg\sqrt{x}}{\sqrt{x}}dx = \int 2tg\,t\,dt = -2\ln|cos\,t| + k = -2\ln|cos\sqrt{x}| + k$

$$t = \sqrt{x} \rightarrow dt = \frac{1}{2\sqrt{x}}dx$$

d) $\int xe^{3x^2}dx = \int \frac{e^t}{6}dt = \frac{e^t}{6} + k = \frac{e^{3x^2}}{6} + k$

$$t = 3x^2 \rightarrow dt = 6x\,dx$$

e) $\int \frac{x}{\sqrt{1-x^4}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{2}arc\,sen\,t + k = \frac{1}{2}arc\,sen\,x^2 + k$

$$t = x^2 \rightarrow dt = 2x\,dx$$

f) $\int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{-1}{2\sqrt{t}}dt = -\sqrt{t} + k = -\sqrt{1-x^2} + k$

$$t = 1-x^2 \rightarrow dt = -2x\,dx$$

g) $\int tg\,2x\,dx = \int \frac{-1}{2t}dt = \frac{-1}{2}\ln|t| + k = \frac{-1}{2}\ln|cos\,2x| + k$

$$t = cos\,2x \rightarrow dt = -2sen\,2x\,dx$$

h) $\int cotg\frac{x}{5}dx = \int 5t\,dt = 5\ln|t| + k = 5\ln\left|sen\frac{x}{5}\right| + k$

$$t = sen\frac{x}{5} \rightarrow dt = \frac{1}{5}cos\frac{x}{5}dx$$

i) $\int \frac{x^4}{\sqrt{1-x^{10}}}dx = \frac{1}{5}\int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{5}arcsen\,t + k = \frac{1}{5}arcsen\,x^5 + k$

$$t = x^5 \rightarrow dt = 5x^4\,dx$$

j) $\int e^x\sqrt{(e^x+1)^3}dx = \int \sqrt{t^3}dt + k = \frac{2}{5}\sqrt{(e^x+1)^5} + k$

$$t = e^x + 1 \rightarrow dt = e^x dx$$

92. Página 290

a) $\int cos^3x\,sen^5x\,dx = \int cos\,x(1-sen^2x)sen^5x\,dx = \int (1-t^2)t^5dt = \int t^5dt - \int t^7dt = \frac{1}{6}t^6 - \frac{1}{8}t^8 + k =$

$= \frac{1}{6}sen^6x - \frac{1}{8}sen^8x + k$

$$t = sen\,x \rightarrow dt = cos\,x\,dx$$

b) $\int \frac{sen^3x}{cos^2x}dx = \int \frac{(1-cos^2x)sen\,x}{cos^2x}dx = \int \frac{t^2-1}{t^2}dt = \int dt - \int \frac{1}{t^2}dt = t + \frac{1}{t} + k = cos\,x + \frac{1}{cos\,x} + k$

$$t = cos\,x \rightarrow dt = -sen\,x\,dx$$

c) $\int sen^3 x\cos^{15} x\,dx = \int (1-\cos^2 x)sen\,x\cos^{15} x\,dx = \int (t^2-1)t^{15}\,dt = \int t^{17}\,dt - \int t^{15}\,dt = \frac{1}{18}t^{18} - \frac{1}{16}t^{16} + k =$

$t = \cos x \rightarrow dt = -sen\,x\,dx$

$= \frac{1}{18}\cos^{18} x - \frac{1}{16}\cos^{16} x + k$

$t = \cos x \rightarrow dt = -sen\,x\,dx$

d) $\int \frac{1}{\cos^3 x \cdot sen\,x}dx = -\int \frac{-sen\,x}{\cos^3 x(1-\cos^2 x)} = -\int \frac{1}{(1-t^2)t^3}dt = \int \left(-\frac{1}{t} - \frac{1}{t^3} + \frac{1}{2(t+1)} + \frac{1}{2(t-1)}\right)dt =$

$= -\ln|t| + \frac{1}{2t^2} + \frac{1}{2}\ln|t+1| + \frac{1}{2}\ln|t-1| + k = -\ln|\cos x| + \frac{1}{2\cos^2 x} + \frac{1}{2}\ln|\cos x+1| + \frac{1}{2}\ln|\cos x-1| + k =$

$= \ln\left|\frac{\sqrt{|1-\cos^2 t|}}{\cos t}\right| + \frac{1}{2\cos^2 t} + k = \ln|tg\,t| + \frac{1}{2\cos^2 t} + k$

93. Página 290

a) $\int \cos^3 dx = \int (1-sen^2 x)\cos x\,dx = \int (1-t^2)dt = t + \frac{t^3}{3} + k = sen\,x + \frac{sen^3 x}{3} + k$

$t = sen\,x \rightarrow dt = \cos x\,dx$

$t = \sec x \rightarrow dt = tg\,x\sec x\,dx$

b) $\int tg^3 x \cdot \sec^3 x\,dx = \int tg\,x(\sec^2 x - 1)\sec^3 x\,dx = \int (t^2-1)t^2 dt = \int (t^4 - t^2)dt = \frac{1}{5}t^5 - \frac{1}{3}t^3 + k = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + k$

c) $\int sen^5 x\,dx = \int sen\,x(1-\cos^2 x)^2 dx = \int -\left(1-t^2\right)^2 dt = \int (-1+2t^2-t^4)dt = \frac{-t^5}{5} + \frac{2t^3}{3} - t + k =$

$t = \cos x \rightarrow dt = -sen\,x\,dx$

$= \frac{-\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + k$

d) $\int \frac{\cos x}{sen x + \cos x}dx = \int \frac{\cos x \cdot \sec^3 x}{(sen\,x + \cos x)\cdot \sec^3 x}dx = \int \frac{\sec^2 x}{tg\,x\sec^2 x + \sec^2 x}dx = \int \frac{1}{tg\,x + 1}dx = \int \frac{1}{(t+1)(t^2+1)}dt =$

$t = tg\,x \rightarrow dt = \sec^2 x\,dx$

$= \int \frac{1-t}{2(t^2+1)}dt + \int \frac{1}{2(t+1)}dt = \frac{1}{2}\int \frac{1}{(t^2+1)}dt - \frac{1}{2}\int \frac{t}{(t^2+1)}dt + \frac{1}{2}\int \frac{1}{(t+1)}dt = \frac{1}{2}arc\,tg\,t - \frac{1}{4}\ln|t^2+1| + \frac{1}{2}\ln|t+1| + k =$

$= \frac{1}{2}x - \frac{1}{4}\ln|tg^2 x + 1| + \frac{1}{2}\ln|tg\,x + 1| + k$

94. Página 290

$t = \cos^2 x - 2sen^2 x \rightarrow dt = -6\cos x\,sen\,x\,dx$

a) $\int \frac{sen x \cdot \cos x}{\cos^2 x - 2sen^2 x}dx = -\frac{1}{6}\int \frac{1}{t}dt = -\frac{1}{6}\ln|t| + k = -\frac{1}{6}\ln|\cos^2 x - 2sen^2 x| + k$

b) $\int \frac{1}{4 - 3\cos^2 x + 5sen^2 x}dx = \int \frac{cosec^2 x}{(4 - 3\cos^3 x + 5sen^2 x)cosec^2 x}dx = \int \frac{cosec^2 x}{4cosec^2 x - 3cotg^2 x + 5}dx = \int \frac{cosec^2 x}{9 + cotg^2 x}dx =$

$= -\int \frac{1}{t^2+9}dt = \frac{1}{3}arc\,tg\left(\frac{t}{3}\right) + k = \frac{1}{3}arc\,tg\left(\frac{cotg\,x}{3}\right) + k$

$t = cotg\,x \rightarrow dt = -cosec^2 x\,dx$

c) $\int \frac{sen^3 x}{\sqrt[3]{\cos x}}\,dx = \int \frac{sen\,x(1-\cos^2 x)}{\sqrt[3]{\cos x}}dx = -\int \frac{1-t^2}{\sqrt[3]{t}}dt = -\int t^{-\frac{1}{3}}dt + \int t^{\frac{5}{3}}dt = -\frac{3}{2}\sqrt[3]{\cos^2 x} + \frac{3\sqrt[3]{\cos^8 x}}{8} + k$

$t = \cos x \rightarrow dt = -sen\,x\,dx$

$t = sen\,x \rightarrow dt = \cos x\,dx$

d) $\int \frac{\cos x}{2sen\,x\cos^2 x + sen^3 x}dx = \int \frac{\cos x}{sen^3 x + 2sen\,x(1-sen^2 x)}dx = \int \frac{\cos x}{2sen\,x - sen^3 x}dx = \int \frac{1}{2t - t^3}dt = \int \left(\frac{1}{2t} - \frac{t}{2(t^2-2)}\right)dt =$

$= \frac{1}{2}\ln|t| - \frac{1}{4}\ln|t^2 - 2| + k = \frac{1}{2}\ln|sen\,x| - \frac{1}{4}\ln|sen^2 x - 2| + k$

95. Página 290

a) $$\int \frac{1+\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx = \int \frac{1+\left(\sqrt[6]{x+1}\right)^3}{1+\left(\sqrt[6]{x+1}\right)^2}dx = \int \frac{1+t^3}{1+t^2}6t^5dt = 6\int \frac{t^8+t^5}{1+t^2}dt = 6\int\left(t^6-t^4+t^3+t^2-t-1+\frac{t}{t^2+1}+\frac{1}{t^2+1}\right)dt =$$

$$t=\sqrt[6]{x+1} \rightarrow dt = \frac{1}{6\sqrt[6]{(x+1)^5}}dx \rightarrow dx = 6t^5dt$$

$$\frac{t^5}{5}+\frac{t^4}{4}+\frac{t^3}{3}-\frac{t^2}{2}-t+\frac{1}{2}\ln|t^2+1|+arctg\,t\Bigg)+k=$$

$$=6\left(\frac{\sqrt[6]{(x+1)}^7}{7}-\frac{\sqrt[6]{(x+1)}^5}{5}+\frac{\sqrt[3]{(x+1)}^2}{4}+\frac{\sqrt{(x+1)}}{3}-\frac{\sqrt[3]{(x+1)}}{2}-\sqrt[6]{(x+1)}+\frac{1}{2}\ln|\sqrt[3]{(x+1)}+1|+arctg\sqrt[6]{(x+1)}\right)+k$$

b) $$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx = \int \frac{t^3}{1+t^2}\cdot 6t^5dt = \int \frac{6t^8}{1+t^2}dt = 6\int\left(t^6-t^4+t^2-1+\frac{1}{t^2+1}\right)dt = 6\left(\frac{t^7}{7}-\frac{t^5}{5}+\frac{t^3}{3}-t+arctg\,t\right)+k$$

$$t=\sqrt[6]{x} \rightarrow dt=\frac{1}{6\sqrt[6]{x^5}}dx \rightarrow dx=6t^5dt$$

$$=6\left(\frac{\sqrt[6]{x^7}}{7}-\frac{\sqrt[6]{x^5}}{5}+\frac{\sqrt{x}}{3}-\sqrt[6]{x}+arctg\sqrt[6]{x}\right)+k$$

c) $$\int \frac{1+x+\sqrt{x+1}}{(x+1)\sqrt[3]{x+1}}dx = \int \frac{t^6+t^3}{t^6\cdot t^2}\cdot 6t^5dt = 6\int (t^3+1)dt = 6\left(\frac{t^4}{4}+t\right)+k = 6\left(\frac{\sqrt[6]{(1+x)^4}}{4}+\sqrt[6]{1+x}\right)+k=$$

$$t=\sqrt[6]{x+1} \rightarrow dt=\frac{1}{6\sqrt[6]{(x+1)^5}}dx \rightarrow dx=6t^5dt$$

$$=6\left(\frac{\sqrt[3]{(1+x)^2}}{4}+\sqrt[6]{1+x}\right)+k$$

d) $$\int \frac{x+\sqrt{x}}{\sqrt{x}+\sqrt[4]{x}}dx = \int \frac{t^4+t^2}{t^2+t}\cdot 4t^3dt = 4\int \frac{t^6+t^4}{t+1}dt = 4\int\left(t^5-t^4+2t^3-2t^2+2t-2+\frac{2}{t+1}\right)dt =$$

$$t=\sqrt[4]{x} \rightarrow dt=\frac{1}{4\sqrt[4]{x^3}}dx \rightarrow dx=4t^3dt$$

$$=4\left(\frac{t^6}{6}-\frac{t^5}{5}+\frac{2t^4}{4}-\frac{2t^3}{3}+\frac{2t^2}{2}-2t+2\ln|t+1|\right)+k=4\left(\frac{\sqrt[4]{x^6}}{6}-\frac{\sqrt[4]{x^5}}{5}+\frac{x}{2}-\frac{2\sqrt[4]{x^3}}{3}+\sqrt{x}-2\sqrt[4]{x}+2\ln|\sqrt[4]{x}+1|\right)+k$$

e) $$\int \frac{1}{\sqrt{x}\left(1+\sqrt[4]{x}\right)}dx = \int \frac{1}{t^2(1+t)}\cdot 4t^3dt = 4\int \frac{t}{1+t}dt = 4\int\left(1+\frac{-1}{1+t}\right)dt = 4t-4\ln|t+1|+k = 4\sqrt[4]{x}-4\ln|\sqrt[4]{x}+1|+k$$

$$t=\sqrt[4]{x} \rightarrow dt=\frac{1}{4\sqrt[4]{x^3}}dx \rightarrow dx=4t^3dt$$

f) $$\int \frac{1}{2+\sqrt{x+1}}dx = \int \frac{1}{2+t}\cdot 2t\,dt = 2\int\left(1+\frac{-2}{t+2}\right)dt = 2t-4\ln|t+2|+k = 2\sqrt{x+1}-4\ln|\sqrt{x+1}+2|+k$$

$$t=\sqrt{x+1} \rightarrow dt=\frac{1}{2\sqrt{x+1}}dx \rightarrow dx=2t\,dt$$

96. Página 290

a) $$\int \frac{1}{\left(1-e^x\right)^2}dx = \int \frac{1}{(1-t)^2}\cdot\frac{1}{t}dt = \int\left(\frac{1}{t}-\frac{1}{t-1}+\frac{1}{(t-1)^2}\right)dt = \ln|t|-\ln|t-1|-\frac{1}{t-1}+k = x-\ln|e^x-1|-\frac{1}{e^x-1}+k$$

$$t=e^x \rightarrow dt=e^xdx \rightarrow dx=$$

$$t=e^x \rightarrow dt=e^xdx$$

b) $$\int \frac{1}{e^x+e^{-x}+1}dx = \int \frac{e^x}{e^{2x}+e^x+1}dx = \int \frac{1}{t^2+t+1}dt = \int \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}dt = \frac{4}{3}\int \frac{1}{\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right)^2+1}dt =$$

$$=\frac{4}{3}\frac{\sqrt{3}}{2}arctg\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right)+k=\frac{2}{\sqrt{3}}arctg\left(\frac{2}{\sqrt{3}}e^x+\frac{1}{\sqrt{3}}\right)+k$$

c) $\int \frac{e^{3x}}{e^{2x}-3e^x+2}dx = \int \frac{t^2}{t^2-3t+2}dt = \int\left(-\frac{1}{t-1}+\frac{4}{t-2}+1\right)dt = -\ln|t-1|+4\ln|t-2|+t+k =$

$t = e^x \rightarrow dt = e^x dx$

$= -\ln|e^x-1|+4\ln|e^x-2|+e^x+k$

d) $\int \frac{1+\sqrt{e^x}}{\left(1-\sqrt[4]{e^x}\right)^2}dx = \int \frac{1+t^2}{(1-t)^2}\cdot\frac{4}{t}dt = 4\int\frac{1+t^2}{t(1-t)^2}dt = 4\int\left(\frac{1}{t}+\frac{1}{1-t}+\frac{t+1}{(1-t)^2}\right)dt = 4\left(\ln|t|-\ln|1-t|+\int\frac{t-1+2}{(1-t)^2}dt\right)+k$

$t = \sqrt[4]{e^x} \rightarrow dt = \frac{\sqrt[4]{e^x}}{4}dx \rightarrow dx = \frac{4}{t}dt$

$= 4\left(\ln|t|-\ln|1-t|+\int\frac{-1}{1-t}dt+\int\frac{2}{(1-t)^2}dt\right)+k = 4\left(\ln|t|-\ln|1-t|+\ln|1-t|+\frac{2}{1-t}\right)+k =$

$= 4\left(\ln|\sqrt[4]{e^x}|+\frac{2}{1-\sqrt[4]{e^x}}\right)+k$

97. **Página 290**

a) $\int 2\,sen\,x\,e^{-\cos x}\,dx = \int 2e^t dt = 2e^t+k = 2e^{-\cos x}+k$

$t = -\cos x \rightarrow dt = sen\,x\,dx$

b) $\int (1-x^2)^{-\frac{3}{2}}dx = \int\frac{\cos t}{(1-sen^2 t)^{\frac{3}{2}}}dt = \int\frac{\cos t}{\cos^3 t}dt = \int\frac{1}{\cos^2 t}dt = tg\,t+k = tg(arc\,sen\,x)+k = \frac{x}{\sqrt{1-x^2}}+k$

$x = sen\,t \rightarrow dx = \cos t\,dt$

c) $\int x^5\sqrt{1-x^2}\,dx = \int sen^5 t\cos^2 t = \int sen\,t(1-\cos^2 t)^2\cos^2 t\,dt = -\int(1-u^2)^2u^2du = -\int\left(u^2-2u^4+u^6\right)du =$

$t = sen\,x \rightarrow dt = \cos x$

$u = \cos t \rightarrow du = -sen\,t\,dt$

$= -\frac{}{3}+\frac{}{5}-\frac{}{7}+k = -\frac{\cos^3 t}{3}+\frac{2\cos^5 t}{5}-\frac{\cos^7 t}{7}+k = \left(-\frac{1}{3}\cos t(1-sen^2 t)+\frac{2}{5}\cos t(1-sen^2 t)^2-\frac{1}{7}\cos t(1-sen^2 t)^3\right)+k =$

$= \sqrt{1-x^2}\cdot\left(-\frac{1-x^2}{3}+\frac{2(1-x^2)^2}{5}-\frac{(1-x^2)^3}{7}\right)+k$

d) $\int\left(1-(2x+1)^2\right)^{-\frac{1}{2}}dx = \frac{1}{2}\int(1-t^2)^{-\frac{1}{2}}dt = \frac{1}{2}\int\frac{1}{\sqrt{1-t^2}}dt = \frac{1}{2}arc\,sen(t)+k = \frac{1}{2}arc\,sen(2x+1)+k$

$t = 2x+1 \rightarrow dt = 2dx$

98. **Página 290**

a) $\int\frac{3x^2-5x+7}{x^3-4x^2+4x}dx = \int\left(\frac{9}{2(x-2)^2}+\frac{5}{4(x-2)}+\frac{7}{4x}\right)dx = \frac{-9}{2(x-2)}+\frac{5}{4}\ln|x-2|+\frac{7}{4}\ln|x|+k$

b) $\int\frac{x}{\sqrt{1-9x^2}}dx = -\frac{1}{18}\int\frac{-18x}{\sqrt{1-9x^2}}dx = -\frac{\sqrt{1-9x^2}}{9}+k$

c) $\int\frac{3x^2-7x+4}{2x-3}dx = \int\left(\frac{3}{2}x-\frac{5}{4}+\frac{1}{4(2x-3)}\right)dx = \frac{3x^2}{4}-\frac{5x}{4}+\frac{1}{8}\ln|2x-3|+k$

d) $\int \frac{2}{9+4x^2}dx = \int \frac{2}{9+(2x)^2}dx = \frac{1}{3}arctg\frac{2x}{3}+k$

e) $\int \frac{5}{\sqrt{1-9x^2}}dx = \frac{5}{3}\int \frac{3}{\sqrt{1-(3x)^2}}dx = \frac{5}{3}arc\,sen\,3x+k$

f) $\int \frac{x}{\sqrt{1+x^2}}dx = \sqrt{1+x^2}+k$

g) $\int \sqrt{1-x^2}dx = \int \sqrt{1-sen^2t}\cos t\,dt = \int \cos^2t\,dt = \cos t\,sen\,t + \int sen^2t\,dt =$

$x = sen\,t \rightarrow dx = \cos t\,dt$

$u = \cos t \rightarrow du = -sen\,t\,dt$
$dv = \cos t \rightarrow v = sen\,t$

$= \cos t\,sen\,t + \int (1-\cos^2t)dt = \cos t\,sen\,t + t - \int \cos^2t\,dt$

$\int \cos^2t\,dt = \cos t\,sen\,t + t - \int \cos^2t\,dt \rightarrow \int \cos^2t\,dt = \frac{1}{2}(\cos t\,sen\,t + t)+k$

$\int \sqrt{1-x^2}dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^2}+arc\,sen\,x}{2}\right)+k$

h) $\int \frac{x}{\sqrt{1+x}}dx = \int \frac{t-1}{\sqrt{t}}dt = \int \sqrt{t}dt - \int \frac{1}{\sqrt{t}}dt = \frac{2}{3}\sqrt{t^3}-2\sqrt{t}+k = \sqrt{1+x}\cdot\left(\frac{2(x+1)}{3}-2\right)+k$

$t = x+1 \rightarrow dt = dx$

99. Página 290

a) $\int \frac{x^3+4x^2-10x+7}{x^3-7x-6}dx = \int \left(1+\frac{2}{x-3}-\frac{5}{x+1}+\frac{7}{x+2}\right)dx =$

$= x + 2\ln|x-3| - 5\ln|x+1| + 7\ln|x+2| + k$

b) $\int \frac{1}{x^2-7x+10}dx = \int \left(\frac{1}{3(x-5)}-\frac{1}{3(x-2)}\right)dx = \frac{1}{3}\ln|x-5| - \frac{1}{3}\ln|x-2|+k$

c) $\int \frac{2^{3x}}{2^x-4}dx = \frac{1}{\ln 2}\int \frac{t^2}{t-4}dt = \frac{1}{\ln 2}\int \left(t+4+\frac{16}{t-4}\right)dt = \frac{1}{\ln 2}\left(\frac{t^2}{2}+4t+16\ln|t-4|\right)+k =$

$t = 2^x \rightarrow dt = 2^x \ln 2dx \rightarrow dx = \frac{dt}{t\ln 2}$

$= \frac{1}{\ln 2}\left(\frac{2^{2x}}{2}+4\cdot 2^x+16\ln|2^x-4|\right)+k$

d) $\int \frac{1}{x\sqrt{1-x}}dx = \int \frac{-2}{1-t^2}dt = \int \frac{-1}{1-t}dt - \int \frac{1}{1+t}dt = \ln|1-t| - \ln|1+t|+k = \ln|1-\sqrt{1-x}| - \ln|1+\sqrt{1-x}|+k$

$t = \sqrt{1-x} \rightarrow dt = \frac{-1}{2\sqrt{1-x}}dx$

e) $\int \frac{x+3}{4x^2+8}dx = \frac{1}{8}\int \frac{2x}{x^2+2}dx + \frac{1}{4}\int \frac{3}{x^2+2}dx = \frac{1}{8}\ln|x^2+2| + \frac{3}{4\sqrt{2}}arctg\frac{x}{\sqrt{2}}+k$

f) $\int \frac{sen\,2x+\cos x}{\cos x}dx = \int \frac{2sen\,x\cos x+\cos x}{\cos x}dx = \int (2sen\,x+1)dx = -2\cos x + x + k$

100. Página 290

a) $\int \operatorname{cosec} x\,dx = \int \frac{1}{\operatorname{sen} x}dx = \int \frac{-1}{1-t^2}dt = \frac{-1}{2}\ln|1+t| + \frac{1}{2}\ln|1-t| + k = -\frac{1}{2}\ln|1+\cos x| + \frac{1}{2}\ln|1-\cos x| + k$

$t = \cos x \rightarrow dt = -\operatorname{sen} x\,dx$

b) $\int \sec x\,dx = \int \frac{1}{\cos x}dx = \int \frac{1}{1-t^2}dt = \frac{1}{2}\ln|1+t| - \frac{1}{2}\ln|1-t| + k = \frac{1}{2}\ln|1+\operatorname{sen} x| - \frac{1}{2}\ln|1-\operatorname{sen} x| + k$

$t = \operatorname{sen} x \rightarrow dt = \cos x\,dx$

c) $\int e^{x^2-5x}(2x-5)\,dx = e^{x^2-5x} + k$

d) $\int \frac{\operatorname{sen} x}{\sqrt{1+\cos x}}dx = -2\sqrt{1+\cos x} + k$

e) $\int \frac{1}{x\sqrt{1-(\ln x)^2}}dx = \operatorname{arc\,sen}(\ln x) + k$

f) $\int \frac{1+e^{3x}}{e^{2x}}dx = \int \frac{1}{e^{2x}}dx + \int e^x dx = \frac{-e^{-2x}}{2} + e^x + k$

g) $\int \sqrt{e^x - 1}\,dx = \int (t+1)\sqrt{t}\,dt = \frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}} + k = \sqrt{e^x-1}\left(\frac{2}{5}(e^x-1)^2 + \frac{2}{3}(e^x-1)\right) + k$

$t = e^x - 1 \rightarrow dt = e^x dx$

h) $\int \frac{\operatorname{sen} 2x}{1+\cos^2 x}dx = \int \frac{2\operatorname{sen} x\cos x}{1+\cos^2 x}dx = -\int \frac{-2\operatorname{sen} x\cos x}{1+\cos^2 x}dx = -2\ln|1+\cos^2 x| + k$

101. Página 290

a) $\int (x-2)e^{3x}dx = \frac{1}{3}(x-2)e^{3x} - \int \frac{e^{3x}}{3}dx = \frac{1}{3}(x-2)e^{3x} - \frac{e^{3x}}{9} + k$

$u = x-2 \rightarrow du = dx$

$dv = e^{3x}dx \rightarrow v = \frac{e^{3x}}{3}$

b) $\int \frac{7}{x\sqrt[3]{\ln x}}dx = \frac{21}{2}\sqrt[3]{(\ln x)^2} + k$

c) $\int \frac{(\ln x)^2 + x}{x}dx = \int \frac{(\ln x)^2}{x}dx + \int dx = \frac{(\ln x)^3}{3} + x + k$

d) $\int (\ln x)^2 dx = (x\ln x - x)\ln x - \int (\ln x - 1)dx = (x\ln x - x)\ln x - x\ln x + 2x + k =$

$= x(\ln x)^2 - 2x\ln x + 2x + k$

$u = \ln x \rightarrow du = \frac{dx}{x}$

$dv = \ln x\,dx \rightarrow v = x\ln x - x$

e) $\int \frac{5e^{2x} - e^x}{e^{2x}-1}dx = \int \frac{5t-1}{t^2-1}dt = \int \frac{2}{t-1}dt + \int \frac{3}{t+1}dt = 2\ln|t-1| + 3\ln|t+1| + k =$

$t = e^x \rightarrow dt = e^x dx$

$= 2\ln|e^x - 1| + 3\ln|e^x + 1| + k$

f) $\int \operatorname{sen}^2 x\,dx = \int \frac{1-\cos 2x}{2}dx = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + k$

g) $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}dx = \frac{x}{2} + \frac{\operatorname{sen} 2x}{4} + k = \frac{x + \operatorname{sen} x\cos x}{2} + k$

h) $\int \operatorname{sen} x\cos x\,dx = \frac{\operatorname{sen}^2 x}{2} + k$

102. Página 290

a) $\int 2^x \cos x\,dx = 2^x \operatorname{sen} x - \ln 2 \int 2^x \operatorname{sen} x\,dx = 2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x - (\ln 2)^2 \int 2^x \cos x\,dx$

$u = 2^x \to du = 2^x \ln 2\,dx$
$dv = \cos x\,dx \to v = \operatorname{sen} x$

$u = 2^x \to du = 2^x \ln 2\,dx$
$dv = \operatorname{sen} x\,dx \to v = -\cos x$

$\int 2^x \cos x\,dx = \frac{2^x \operatorname{sen} x + \ln 2 \cdot 2^x \cos x}{1 + (\ln 2)^2}$

b) $\int \frac{\ln x + 3}{x(\ln x - 1)}dx = \int \frac{t+3}{t-1}dt = t + 4\ln|t-1| + k = \ln|x| + 4\ln|\ln x - 1| + k$

$t = \ln x \to dt = \frac{dx}{x}$

c) $\int \cos(\ln x)dx = \int e^t \cos t\,dt = e^t \operatorname{sen} t - \int e^t \operatorname{sen} t\,dt = e^t \operatorname{sen} t + e^t \cos t - \int e^t \cos t\,dt$

$t = \ln x \to dt = \frac{dx}{x}$

$u = e^t \to du = e^t\,dt$
$dv = \cos t\,dt \to v = \operatorname{sen}$

$u = e^t \to du = e^t\,dt$
$dv = \operatorname{sen} t\,dt \to v = -\cos t$

$\int \cos(\ln x)dx = \frac{e^t \operatorname{sen} t + e^t \cos t}{2} + k = \frac{x \operatorname{sen}(\ln x) + x\cos(\ln x)}{2} + k$

d) $\int \operatorname{sen}\sqrt{x}\,dx = 2\int t \operatorname{sen} t\,dt = 2\left(-t\cos t + \int \cos t\,dt\right) = -2t\cos t + 2\operatorname{sen} t + k = -2\sqrt{x}\cos\sqrt{x} + 2\operatorname{sen}\sqrt{x} + k$

$t = \sqrt{x} \to dt = \frac{dx}{2\sqrt{x}}$

$u = t \to du = dt$
$dv = \operatorname{sen} t\,dt \to v = -\cos t$

e) $\int \operatorname{sen}^2 x \cos x\,dx = \frac{\operatorname{sen}^3 x}{3} + k$

f) $\int \operatorname{sen}^3 x \cos x\,dx = \frac{\operatorname{sen}^4 x}{4} + k$

g) $\int \operatorname{sen}^3 x\,dx = \int (1 - \cos^2 x)\operatorname{sen} x\,dx = \int \operatorname{sen} x\,dx - \int \cos^2 x \operatorname{sen} x\,dx = -\cos x + \frac{\cos^3 x}{3} + k$

h) $\int \cos^3 x\,dx = \int (1 - \operatorname{sen}^2 x)\cos x\,dx = \int \cos x\,dx - \int \operatorname{sen}^2 x \cos x\,dx = \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + k$

103. Página 291

a) $\int x \operatorname{sen}(\ln x)dx = \frac{x^2}{2}\operatorname{sen}(\ln x) - \frac{1}{2}\int x\cos(\ln x)dx = \frac{x^2}{2}\operatorname{sen}(\ln x) - \frac{x^2}{4}\cos(\ln x) - \frac{1}{4}\int x \operatorname{sen}(\ln x)dx$

$u = \operatorname{sen}(\ln x) \to du = \frac{\cos(\ln x)}{x}d$
$dv = x\,dx \to v = \frac{x^2}{2}$

$u = \cos(\ln x) \to du = \frac{-\operatorname{sen}(\ln x)}{x}dx$
$dv = x\,dx \to v = \frac{x^2}{2}$

$\frac{5}{4}\int x \operatorname{sen} x(\ln x)dx = \frac{x^2}{2}\operatorname{sen}(\ln x) - \frac{x^2}{4}\cos(\ln x) + k \to \int x \operatorname{sen} x(\ln x)dx = \frac{2x^2 \operatorname{sen}(\ln x) - x^2\cos(\ln x)}{5} + k$

b) $\int \operatorname{tg} x \sec^2 x\,dx = \int \frac{\operatorname{sen} x}{\cos^3 x}dx = \frac{1}{2\cos^2 x} + k$

c) $\int \frac{\cos^3 x}{\operatorname{sen} x}dx = \int \frac{1-t^2}{t}dt = \ln|t| - \frac{t^2}{2} + k = \ln|\operatorname{sen} x| - \frac{\operatorname{sen}^2 x}{2} + k$

$t = \operatorname{sen} x \to dt = \cos x\,dx$

d) $\int(\cos^2 x-\operatorname{sen} x\cos^2 x)dx=\int\cos^2 x\,dx-\int\operatorname{sen} x\cos^2 x\,dx=\int\frac{1+\cos 2x}{2}dx-\int\operatorname{sen} x\cos^2 x\,dx=$

$=\frac{x}{2}+\frac{\operatorname{sen} 2x}{4}-\frac{\cos^3 x}{3}+k$

e) $\int\frac{\cos x-\operatorname{sen} x}{2}dx=\int\frac{\cos x}{2}dx-\int\frac{\operatorname{sen} x}{2}dx=\frac{\operatorname{sen} x+\cos x}{2}+k$

f) $\int\frac{\cos^2 x\operatorname{sen} x+\cos x\operatorname{sen}^2 x}{\operatorname{sen} x}dx=\int(\cos^2 x+\cos x\operatorname{sen} x)dx=\int\cos^2 x\,dx+\int\cos x\operatorname{sen} x\,dx=$

$=\int\frac{1+\cos 2x}{2}dx+\int\cos x\operatorname{sen} x\,dx=\frac{x}{2}+\frac{\operatorname{sen} 2x}{4}+\frac{\operatorname{sen}^2 x}{2}+k$

104. Página 291

a) $\int x^3\sqrt{2x+1}\,dx=\frac{1}{2}\int\left(\frac{t-1}{2}\right)^3\cdot\sqrt{t}\,dt=\frac{1}{16}\int(t-1)^3\sqrt{t}\,dt=\frac{1}{16}\int(t^{7/2}-3t^{5/2}+3t^{3/2}-t^{1/2})dt=$

$t=2x+1\rightarrow dt=2dx$

$=\frac{1}{16}\left(\frac{2}{9}t^{9/2}-\frac{6}{7}t^{7/2}+\frac{6}{5}t^{5/2}-\frac{2}{3}t^{3/2}\right)+k=\frac{1}{16}\left(\frac{2}{9}(2x+1)^{9/2}-\frac{6}{7}(2x+1)^{7/2}+\frac{6}{5}(2x+1)^{5/2}-\frac{2}{3}(2x+1)^{3/2}\right)+k$

b) $\int\frac{-2x+6}{x^3-2x^2-x+2}dx=\int\left(-\frac{2}{x-1}+\frac{4}{3(x+1)}+\frac{2}{3(x-2)}\right)=-2\ln|x-1|+\frac{4}{3}\ln|x+1|+\frac{2}{3}\ln|x-2|+k$

c) $\int\frac{\operatorname{sen} 2x}{\sqrt{1+\cos 2x}}dx=-\frac{1}{2}\int\frac{1}{\sqrt{t}}dt=-\sqrt{t}+k=-\sqrt{1+\cos 2x}+k$

$t=1+\cos 2x\rightarrow dt=-2\operatorname{sen} 2x\,dx$

d) $\int\frac{1}{x^4\sqrt{x^2-1}}dx=\int\frac{\operatorname{tg} t\sec t}{\sec^4 t\cdot\sqrt{\sec^2 t-1}}dt=\int\frac{\operatorname{tg} t\sec t}{\sec^4 t\cdot\operatorname{tg} t}dt=\int\frac{1}{\sec^3 t}dt=\int\cos^3 t\,dt=\int(1-\operatorname{sen}^2 x)\cos x\,dx=$

$x=\sec t\rightarrow dx=\operatorname{tg} t\sec t\,dt$

$=\int\cos x\,dx-\int\operatorname{sen}^2 x\cos x\,dx=\operatorname{sen} x-\frac{\operatorname{sen}^3 x}{3}+k$

105. Página 291

a) $\int 4\operatorname{sen} 3x\operatorname{sen} 2x\,dx=4\int\operatorname{sen} 3x\operatorname{sen} 2x\,dx=4\int\frac{1}{2}(\cos(-x)-\cos(5x))dx=-2\operatorname{sen}(-x)-\frac{2}{5}\operatorname{sen}(5x)+k$

b) $\int\frac{(2+x)^2}{x(4+x^2)}dx=\int\left(\frac{4}{x^2+4}+\frac{1}{x}\right)dx=\int\frac{1}{\left(\frac{x}{2}\right)^2+1}dx+\int\frac{1}{x}dx=2\operatorname{arctg}\left(\frac{x}{2}\right)+\ln|x|+k$

c) $\int-3\operatorname{sen} 2x\cos x\,dx=-3\int\frac{1}{2}(\operatorname{sen} x+\operatorname{sen} 3x)dx=\frac{3}{2}\cos x+\frac{1}{2}\cos 3x+k$

d) $\int\frac{3x^2+5}{2x^2+4}dx=\int\left(\frac{3}{2}-\frac{1}{2(x^2+2)}\right)dx=\int\frac{3}{2}dx-\frac{1}{4}\int\frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}dx=\frac{3}{2}x-\frac{\sqrt{2}}{4}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right)+k$

106. Página 291

a) $\int \frac{1}{\sqrt{x+2}+\sqrt{x-2}}dx = \int \frac{1}{4}\left(\sqrt{x+2}-\sqrt{x-2}\right)dx = \frac{1}{6}\sqrt{(x+2)^3} - \frac{1}{6}\sqrt{(x-2)^3} + k$

b) $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx = \int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{(1+x^2)(1-x^2)}}dx = \int \frac{1}{\sqrt{1+x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx =$

$= arc\,senh(x) + arc\,sen(x) + k$

c) $\int \frac{x^4+5\sqrt[3]{x}-3x\sqrt{x}-2}{4x}dx = \frac{1}{4}\int\left(x^3+5x^{-\frac{2}{3}}-3\sqrt{x}-2x^{-1}\right)dx = \frac{1}{4}\left(\frac{1}{4}x^4+15\sqrt[3]{x}-2\sqrt{x^3}-2\ln|x|\right)+k$

d) $\int\left(x-x^{-3}\right)\sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int\left(x-x^{-3}\right)\cdot x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}\cdot x^{\frac{1}{8}}\,dx = \int\left(x-x^{-3}\right)\cdot x^{\frac{7}{8}}\,dx = \int\left(x^{\frac{15}{8}}-x^{-\frac{17}{8}}\right)dx = \frac{8}{23}x^{\frac{23}{8}}+\frac{8}{9}x^{-\frac{9}{8}}+k$

$= \frac{8}{23}\sqrt[8]{x^{23}} + \frac{8}{9\sqrt[8]{x^9}} + k$

107. Página 291

a) $\int f(x)dx = \int \frac{3}{\sqrt[3]{x^2}}dx = 3\int x^{-\frac{2}{3}}dx = 9\sqrt[3]{x}+k$

b) $\int g(x)dx = \int \frac{x+5}{x^2+x-2}dx = \int\left(\frac{2}{x-1}-\frac{1}{x+2}\right)dx = 2\ln|x-1| - \ln|x+2| + k$

c) $\int h(x)dx = \int \frac{\cos x}{1+sen^2 x}dx = \int \frac{1}{1+t^2}dt = arc\,tg\,t + k = arc\,tg(sen\,x)+k$

$t = sen\,x \rightarrow dt = \cos x\,dx$

d) $\int i(x)dx = \int \frac{1}{4+x^2}dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx = \frac{1}{2}arc\,tg\left(\frac{x}{2}\right)+k$

108. Página 291

a) $\int \frac{sen\,5x}{\cos^2 5x}dx = -\frac{1}{5}\int \frac{1}{t^2}dt = \frac{1}{5t}+k = \frac{1}{5\cos 5x}+k$

$t = \cos 5x \rightarrow dt = -5sen\,5x\,dx$

b) $\int \frac{3x+2}{x^2+8x+7}dx = \int\left(\frac{19}{6(x+7)}-\frac{1}{6(x+1)}\right)dx = \frac{19}{6}\ln|x+7| - \frac{1}{6}|x+1|+k$

c) $\int x^2\cdot\sqrt{3x^3+7}\,dx = \frac{1}{9}\int\sqrt{t}+k = \frac{1}{9}\cdot\frac{2}{3}\cdot t^{\frac{3}{2}}+k = \frac{2}{27}\sqrt{(3x^3+7)^3}+k$

$t = 3x^3+7 \rightarrow dt = 9x^2\,dx$

d) $\int\left(\frac{\sqrt{x}}{3x}-\frac{5x}{\sqrt[3]{x}}\right)dx = \int\left(\frac{1}{3}x^{-\frac{1}{2}}-5x^{\frac{2}{3}}\right)dx = \frac{2}{3}x^{\frac{1}{2}}-3x^{\frac{5}{3}}+k = \frac{2\sqrt{x}}{3}-5\sqrt[3]{x^5}+k$

109. **Página 291**

a) $\int \frac{3e^x+e^{3x}}{e^x}dx=\int \frac{3e^x}{e^x}dx+\int \frac{e^{3x}}{e^x}dx=3\int dx+\int e^{2x}dx=3x+\frac{1}{2}e^{2x}+k$

b) $\int \frac{e^x}{1-e^{2x}}dx=\int \frac{1}{1-t^2}dt=\int\left(\frac{1}{2(t+1)}-\frac{1}{2(t-1)}\right)dt=\frac{1}{2}\ln|t+1|-\frac{1}{2}\ln|t-1|+k=\frac{1}{2}\ln|e^x+1|-\frac{1}{2}\ln|e^x-1|+k$

$t=e^x \rightarrow dt=e^x dx$

c) $\int \frac{x-1}{\sqrt{2x}-\sqrt{x+1}}dx=\int \frac{(x-1)\left(\sqrt{2x}+\sqrt{x+1}\right)}{x-1}dx=\int\left(\sqrt{2x}+\sqrt{x+1}\right)dx=\frac{2\sqrt{2}}{3}\sqrt{x^3}+\frac{2}{3}\sqrt{(x+1)^3}+k$

110. **Página 291**

Si $a=0$, entonces $\int \frac{a}{\sqrt{a^2-x^2}}dx=k$.

Si $a\neq 0$, entonces $\int \frac{a}{\sqrt{a^2-x^2}}dx=\int \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}dx=a\cdot \text{arc sen}\left(\frac{x}{a}\right)+k$.

111. **Página 291**

a) $\int \frac{1+(\ln x)^3}{x\left(\ln^4 x+\ln^2 x\right)}dx=\int \frac{1+t^3}{t^4+t^2}dt=\int\left(\frac{t-1}{t^2+1}+\frac{1}{t^2}\right)dt=\int\left(\frac{t}{t^2+1}-\frac{1}{t^2+1}+\frac{1}{t^2}\right)dt=\frac{1}{2}\ln|t^2+1|-\text{arc tg}\, t-\frac{1}{t}+k=$

$t=\ln x \rightarrow dt=\frac{1}{x}dx$

$=\frac{1}{2}\ln|(\ln x)^2+1|-\text{arc tg}(\ln x)-\frac{1}{\ln x}+k$

b) $\int e^x\left[e^x\cdot sen\left(e^x\right)\right]dx=\int t\, sen\, t\, dt=-t\cos t-\int \cos t\, dt=-t\cos t-sen\, t+k=-e^x\cos e^x-sen\, e^x+k$

$t=e^x \rightarrow dt=e^x dx$

112. **Página 291**

Si $a\neq 2$:

$\int \frac{3dx}{x^2-(a+2)x+2a}=\int \frac{3dx}{(x-a)(x-2)}=-\frac{3}{2-a}\int \frac{dx}{x-a}+\frac{3}{2-a}\int \frac{dx}{x-2}=-\frac{3}{2-a}\ln|x-a|+\frac{3}{2-a}\ln|x-2|+k$

Si $a=2$:

$\int \frac{3}{(x-2)^2}dx=\int 3(x-2)^{-2}dx=\frac{-3}{x-2}+k$

113. **Página 291**

$\int\left(1-\cos^2 x\right)\cdot sen\, 2x\cdot e^{\cos^2 x}dx=\int\left(1-\cos^2 x\right)\cdot 2\, sen\, x\cos x\cdot e^{\cos^2 x}dx=-\int(1-t)e^t dt=-\int e^t dt+\int te^t dt=$

$t=\cos^2 x \rightarrow dt=-2\cos x\, sen\, x\, dx$

$u=t \rightarrow du=dt$
$dv=e^t dt \rightarrow v=e^t$

$=-e^t+\left[te^t-\int e^t dt\right]=-e^t+te^t-e^t+k=e^{\cos^2 x}(\cos^2 x-2)+k$

114. Página 291

$$F(x)=\int f(x)dx=\int x\cdot e^{2x}dx=\frac{1}{2}xe^{2x}-\frac{1}{2}\int e^{2x}dx=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+k$$

$$F(0)=2\rightarrow-\frac{1}{4}+k=2\rightarrow k=\frac{9}{4}$$

La función que cumple estas condiciones es: $F(x)=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+\frac{9}{4}$

115. Página 291

a) $f(x)=\int f'(x)dx=\begin{cases}f_1(x)=\int(3x^2-2)dx & \text{si} \quad x\le 1\\ f_2(x)=\int(1+\ln x)dx & \text{si} \quad x>1\end{cases}$

$$f_1(x)=\int(3x^2-2)dx=x^3-2x+k\rightarrow f_1(-1)=2\rightarrow$$

$$-1+2+k=2\rightarrow k=1\rightarrow f_1(x)=x^3-2x+1$$

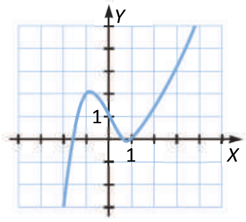
$$f_2(x)=\int(1+\ln x)dx=x+\int\ln x dx=$$

$$=x+x\ln x-\int dx=x+x\ln x-x=x\ln x+k$$

F debe ser continua en *x* = 1, entonces: $f_1(1)=f_2(1)\rightarrow 1-2+1=1\cdot\ln 1+k\rightarrow k=0\rightarrow f_2(x)=x\ln x$

$$f(x)=\int f'(x)dx=\begin{cases}f_1(x)=x^3-2x+1 & \text{si} \quad x\le 1\\ f_2(x)=x\ln x & \text{si} \quad x>1\end{cases}$$

b)



116. Página 291

$$f(x)=\int f'(x)dx=\begin{cases}f_1(x)=\int\frac{-1}{\sqrt{-x}}dx & \text{si} \quad x\le -1\\ f_2(x)=\int(5x^4-6x^2)dx & \text{si} \quad x>-1\end{cases}$$

$$f_1(x)=\int\frac{-1}{\sqrt{-x}}dx=2\sqrt{-x}+k$$

$$f_2(x)=\int(5x^4-6x^2)dx=x^5-2x^3+k\rightarrow f_2(2)=15\rightarrow 32-16+k=15\rightarrow k=-1\rightarrow f_2(x)=x^5-2x^3-1$$

F debe ser continua en x = –1, entonces: $f_1(-1)=f_2(-1)\rightarrow 2+k=0\rightarrow k=-2\rightarrow f_1(x)=2\sqrt{-x}-2$

$$f(x)=\begin{cases}f_1(x)=2\sqrt{-x}-2 & \text{si} \quad x\le -1\\ f_2(x)=x^5-2x^3-1 & \text{si} \quad x>-1\end{cases}$$

117. Página 291

$$\int\frac{-3}{(3x+a)^2}dx=-\int 3(3x+a)^{-2}dx=\frac{1}{3x+a}+k$$

a) Para que *y* = 4 sea asíntota, *k* debe valer 4.

Para que el eje de abscisas (*y* = 0) sea asíntota, *k* debe valer 0.

b) Para que *x* = 1 sea asíntota, *a* debe valer –3.

Para que el eje de ordenadas (*x* = 0) sea asíntota, *a* debe valer 0.

MATEMÁTICAS EN TU VIDA

1. Página 292

El beneficio viene dado por: $R(x) = 2300 - (x - 50)^2$

Vendiendo 30 pares: $R(30) = 2300 - (30 - 50)^2 = 1900$

Vendiendo 25 pares: $R(25) = 2300 - (25 - 50)^2 = 1675$

2. Página 292

Con la venta de 50 pares de zapatillas se obtiene el beneficio máximo, por lo que si los precios no varían, los beneficios empezarían a disminuir.

Si se venden menos de 50 pares, la empresa obtiene beneficios, pero no llegan al beneficio máximo.

3. Página 292

Veamos para qué valores de *x* la función de beneficio es positiva. Para ello, buscaremos los puntos en los que dicha función se anula:

$$R(x) = 0 \to 2300 - (x - 50)^2 = 0 \Leftrightarrow \begin{cases} x_1 = -10(\sqrt{23} - 5) \simeq 2,04 \\ x_2 = 10(5 + \sqrt{23}) \simeq 97,96 \end{cases}$$

La función de beneficio se anula en $x = 2,04$ y en $x = 97,96$. Comprobemos que en valores intermedios la función es positiva, tomando, por ejemplo, $x = 10$.

$R(10) = 700 > 0$

Tenemos, por tanto, que la función de beneficio toma valores positivos en el intervalo $(2,04;\ 97,96)$, pero como estamos trabajando con pares de zapatos, los valores deben ser enteros, por lo que diremos que obtenemos beneficio en el intervalo $[3, 97]$.

4. Página 292

Como ya hemos hallado el intervalo en el que se obtiene beneficio, el mínimo beneficio se obtendrá en alguno de los extremos del intervalo. Veamos en cuál:

$R(3) = 91 = R(97)$

En ambos extremos se obtiene el mismo beneficio, que es de 91 €.